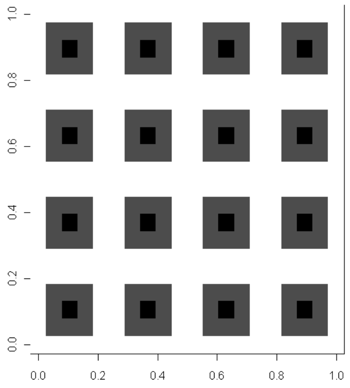

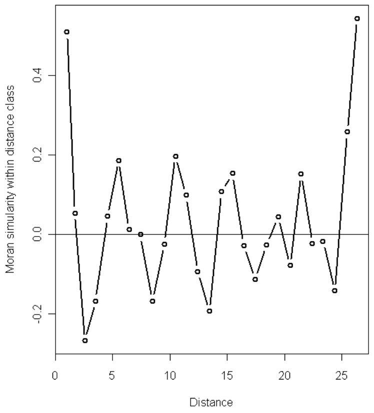

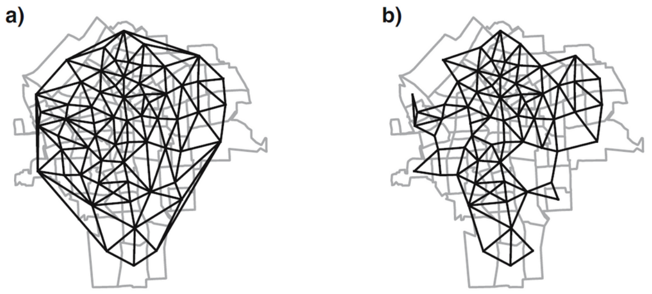

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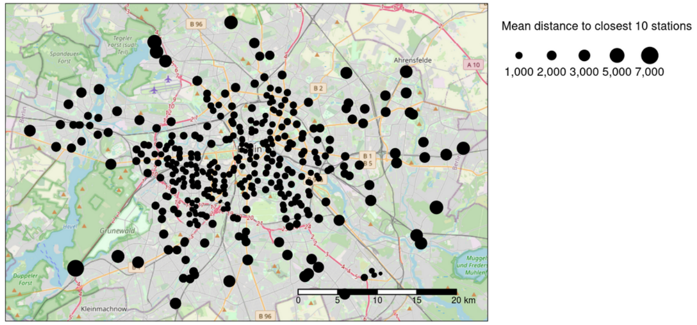
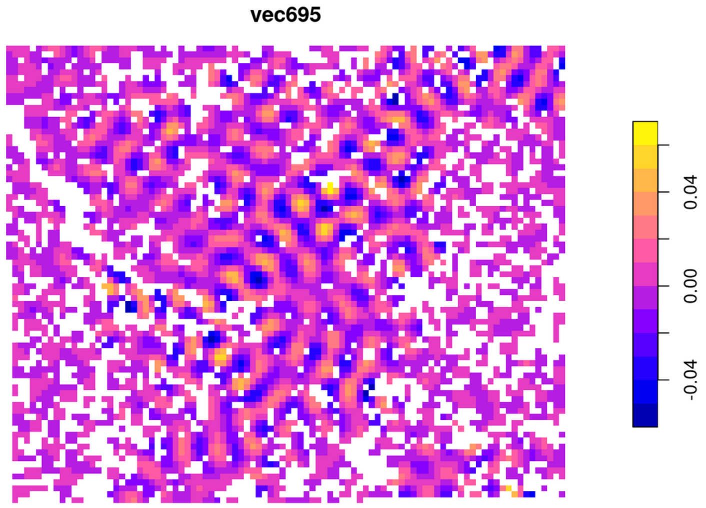
Spatial analysis



Sven Lautenbach

Urban Data Science Workshop

19.-23.10.2020



UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386
GIScience
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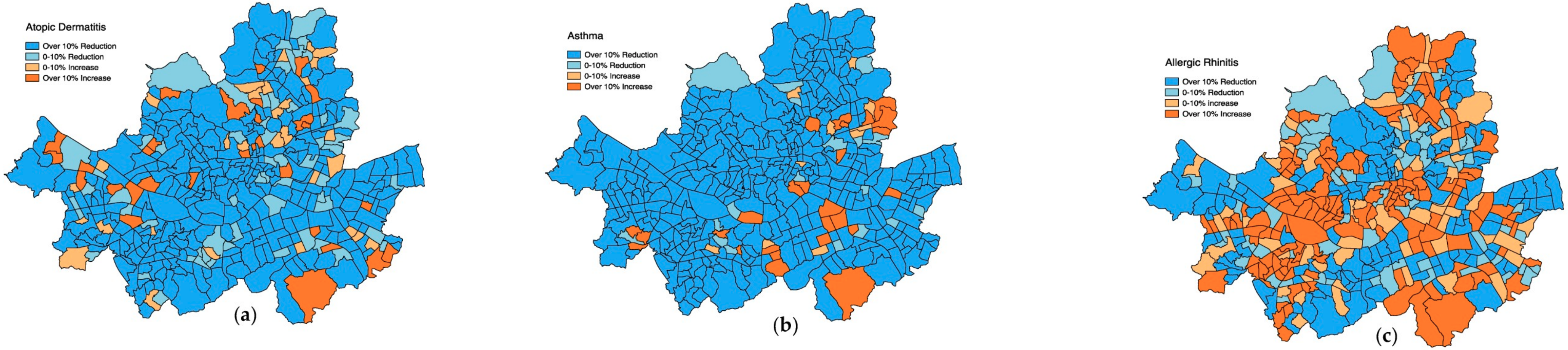
KTS

Overview

- Intro
- Spatial explorative analysis
- Defining neighborhoods and assigning weights
- Detecting spatial autocorrelation
- Spatial clusters
- Dealing with it in regression context
  - Scoping the problem
  - Spatial Eigenvector Mapping
  - Spatial lag and spatial error model
  - Other approaches

What is special about space?

- Tobler‘s first law of geography "everything is related to everything else, but near things are more related than distant things."



Kim et al. A Closer Look at the Bivariate Association between Ambient Air Pollution and Allergic Diseases: The Role of Spatial AnalysisInt J Environ Res Public Health. 2018 Aug; 15(8): 1625

Important tasks in spatial statistics

- Point pattern analysis
  - Analysis of spatial configuration of a population (not a sample)
  - Requires point data
- Interpolation
  - Estimation of surfaces from a sample of observation
- **Spatial explorative analysis**
  - Analysis of **spatial autocorrelation**, hotspots and coldspots
  - **Spatial Cluster analysis**
  - Geographically weighted regression
- **Regression analysis under incorporation of spatial autocorrelation**

Short history of spatial statistics

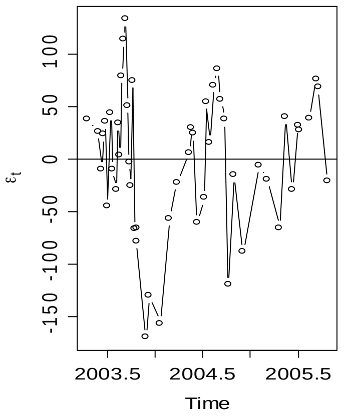
- Development independent in different disciplines:
  - Geosciences especially for mining → Geostatistics (Kriging), e.g. Cressie
  - Ecology, e.g. Legendre, Levin
  - Spatial econometrics, e.g. Anselin
  - Geography, e.g. Griffith, Haining
- Therefore related concepts with varying terminology

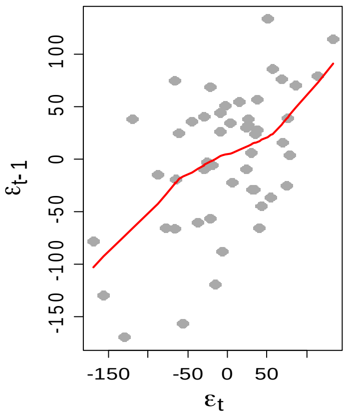
Spatial explorative analysis

- Visual interpretation of spatial data is e.g. complicated due to the necessary classification of continuous data
- It is not always obvious if a spatial pattern occurs in the data
  - Clustered
  - Regular
  - Random
- Spatial autocorrelation is a way to quantify this
- A number of tools exist that quantified the relationship
- In difference to point pattern attribute values have a higher importance
  - Not only geometry but distribution of z-values in space
  - Data might be points, polylines or polygons

Autocorrelation

- Much classic statistical theory assumes that errors are independent and identically distributed (iid), often conforming to a Gaussian distribution
  - Simplifies equations since covariation terms can be set to zero
- However, errors might be structured, violating the simplifying assumptions
- Hierarchical structure, e.g. by unaccounted effects of groups
- Temporal autocorrelation
- Spatial autocorrelation





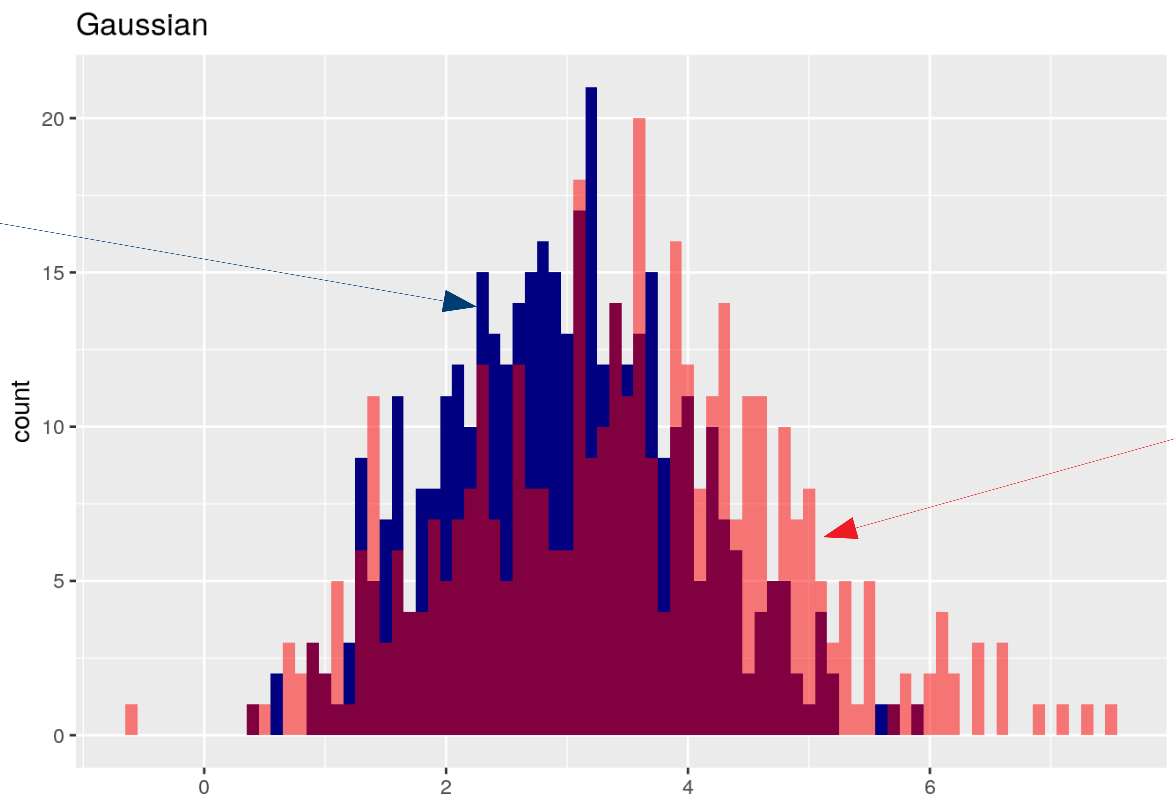
Effects of autocorrelation

- Violating assumptions
- Increasing variance for positive spatial autocorrelation, decreasing variance for negative spatial autocorrelation
  - Estimated standard errors are too small (or too big for negative sac) which effects p-values
  - Mixing up model selection
- Increase or decrease correlation coefficients
- Sample sizes goes down (for positive sac)
  - Effecting standard errors and p-values and model selection procedures

Effects of positive autocorrelation

•

No sac



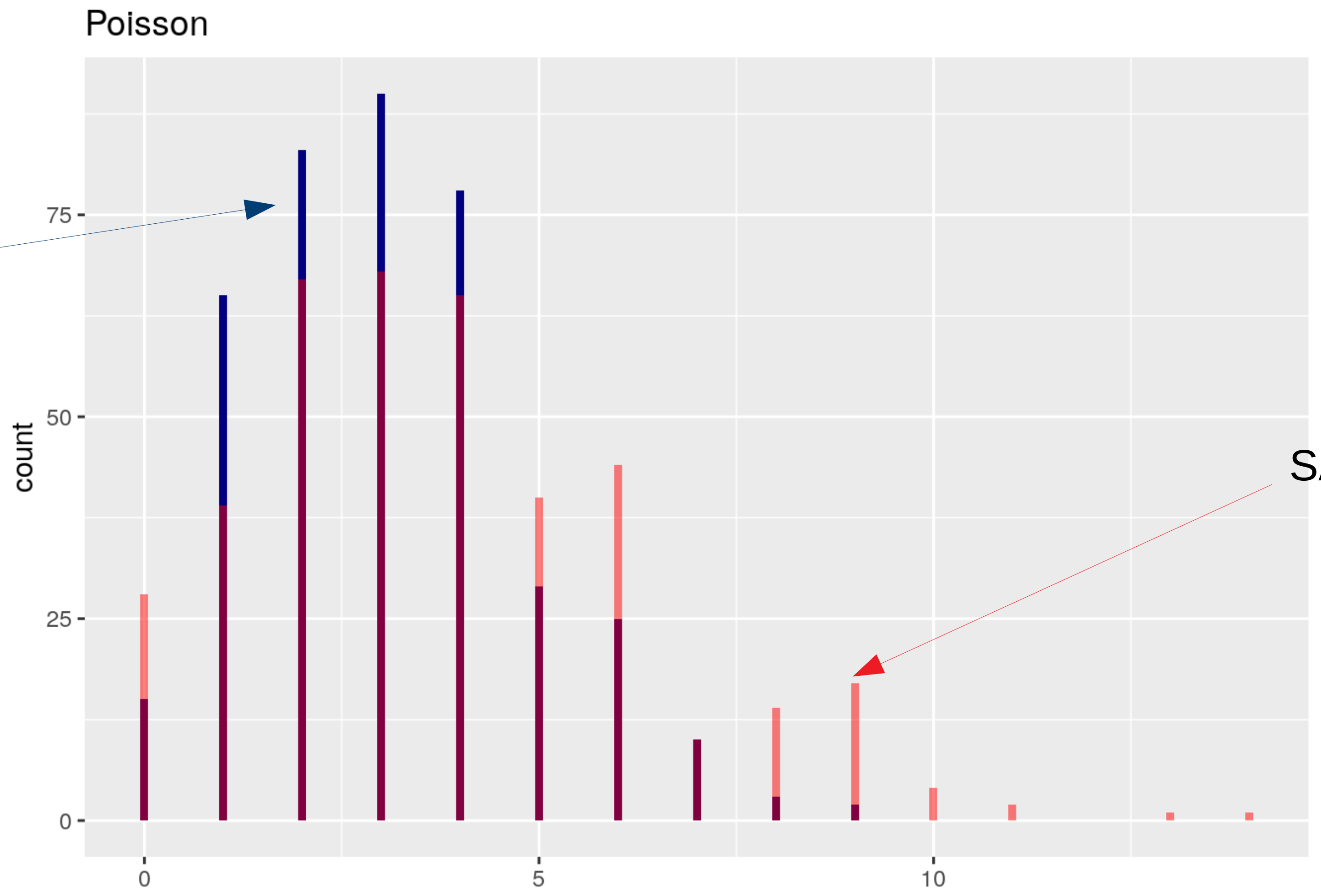
SAC

Increasing variance
Tails become heavier
Preservation of the mean

Effects of positive autocorrelation

-

No sac



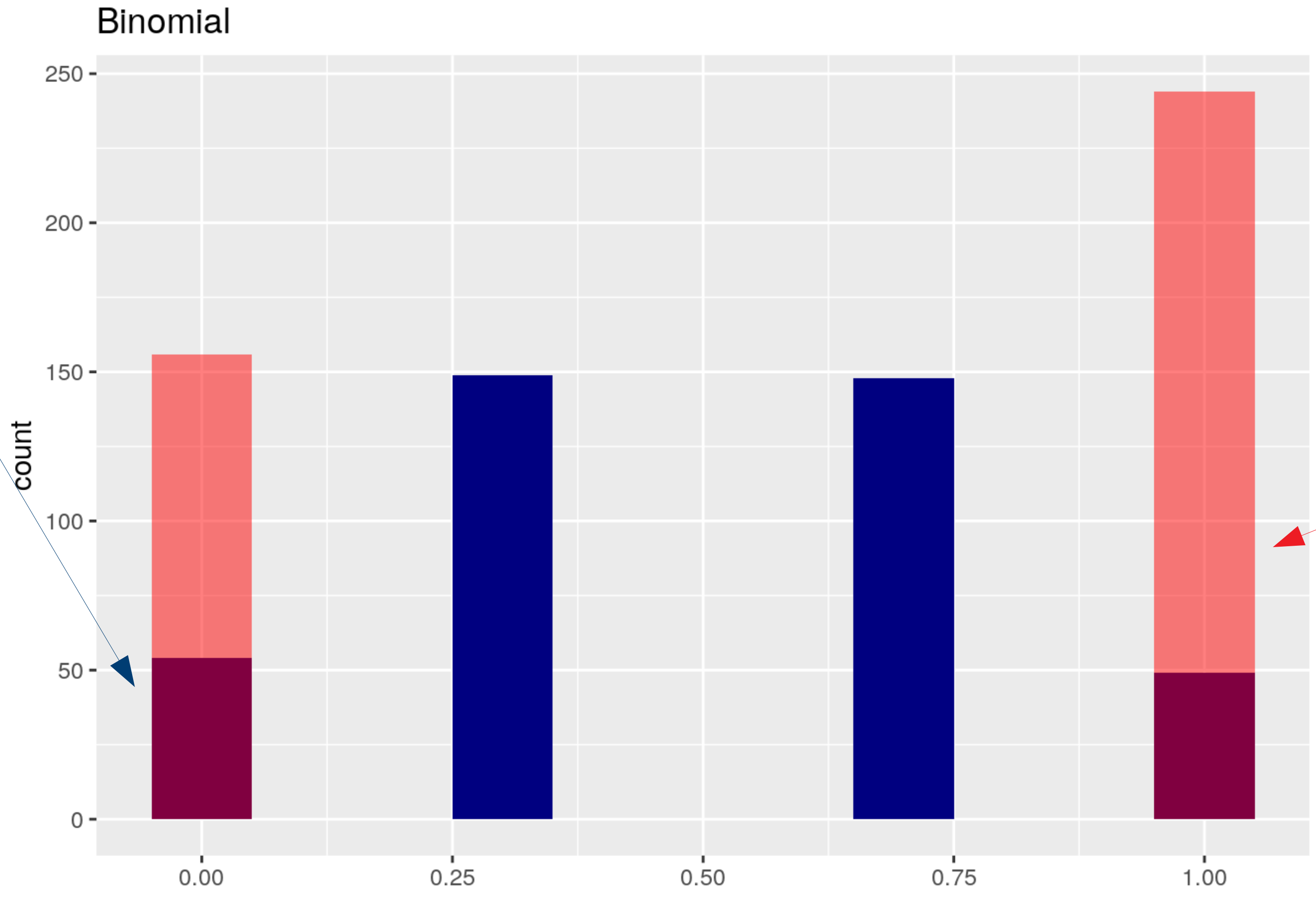
SAC

Increasing variance
More zeros and outliers
Preservation of the mean

Effects of positive autocorrelation

•

No sac



SAC

Distribution moving to the
All-or nothing situation
Preservation of the mean

Mechanisms behind spatial autocorrelation

- **Induced spatial dependence**
  function dependency of the response on spatially structured predictors
  e.g. ecological niche for mosquitos of Aedes type, distribution of charging stations follows population density
- **True autocorrelation**
  functional dependency between the response and adjacent response values, e.g. infectious diseases, distribution of charging stations depends on charging stations in adjacent units
- **Historical dynamics**
  Past events have led to a spatial structure that influences the response
  e.g. infrastructure (railroads, highways) led to spatially structured path dependencies

Reasons for spatial autocorrelation

- The response is autocorrelated
  - High densities of charging stations in one neighborhood might reduce number in adjacent neighborhoods (negative autocorrelation, spill over effect)
- A predictor with a spatial structure is missing which might lead to autocorrelation of the residuals
- We cannot distinguish between the two cases from the residuals
- From a theoretical perspective we might be able to develop a hypothesis on the reason for the presence of spatial autocorrelation

Spatial autocorrelation

Complete independence

$$Y_i = \varepsilon_i, \varepsilon_i \approx N(0, \sigma_\varepsilon^2)$$

Spatial independence

$$y_i = \beta z_i + \varepsilon_i$$
$$z_i = \xi_i$$
$$\xi_i \approx N(0, \sigma_\xi^2)$$

Inherent autoregressive

$$y_i = \rho y_{i-1} + \varepsilon_i$$
$$-1 \leqslant \rho \leqslant 1$$

Induced autoregressive

$$y_i = \beta z_i + \varepsilon_i$$
$$z_i = \rho_z z_i + \xi_i$$

Doubly autoregressive

$$y_i = \beta z_i + \rho_y y_{i-1} + \varepsilon_i$$
$$z_i = \rho z_i + \xi_i$$
$$\xi_i \approx N(0, \sigma_\xi^2)$$

Representation of autocorrelation

- Autocorrelation can be e.g. represented in the variance-covariance matrix of the error term

$$Y = X\beta + \varepsilon$$
$$\varepsilon \approx N\left(0, I\sigma^2\right)$$

Independent errors

$$Y = X\beta + \varepsilon$$
$$\varepsilon \approx N\left(0, \sum \sigma^2\right)$$

Structured errors

- The variance-covariance matrix of the error term is assumed (and tested) to follow a specific structure:
  - Correlation in groups: covariance only for members of the same group
  - Temporal auto-correlation: covariance depends on temporal lag
  - Spatial auto-correlation: covariance depends on spatial lag / neighborhood definition

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Defining spatial relationship
Neighbors and spatial weights

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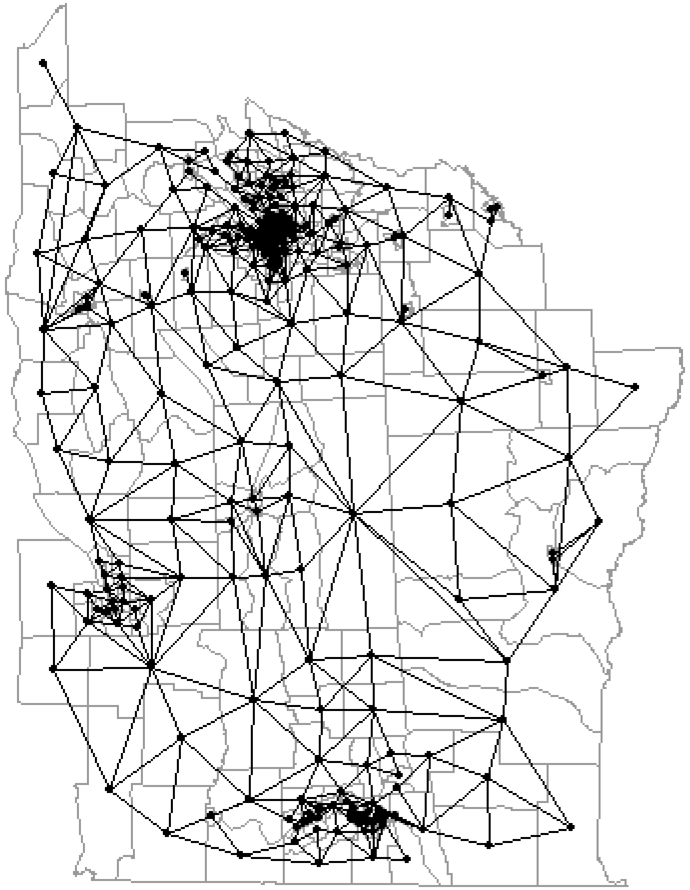
Defining neighborhoods

- A neighborhood (or contiguity) matrix **C** represents if pairs of spatial features are to be considered as neighbors or not
- A spatial weight **W** matrix is a weighted form of such a neighborhood matrix
- **W** represents the possible spatial interactions for the selected neighborhood+ weighting approach

Neighborhood

- Spatial autocorrelation depends on a defined neighborhood
- What is a neighbor?
- Several approaches possible
- Polygon or raster data: contiguity relationship
  - Polygons that share a border are neighbors
  - Rook or queen



Queen and Rook neighborhood

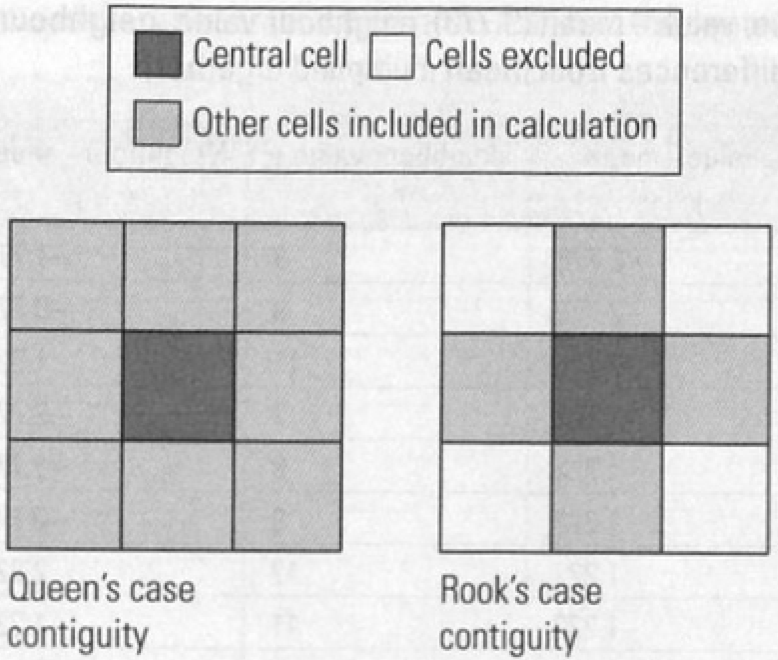


Figure 4.9 Queen's case contiguity and rook's case contiguity.

Source: Lloyd, C. (2010). Spatial data analysis: an introduction for GIS users. Oxford university press.

Queen and Rook neighborhood

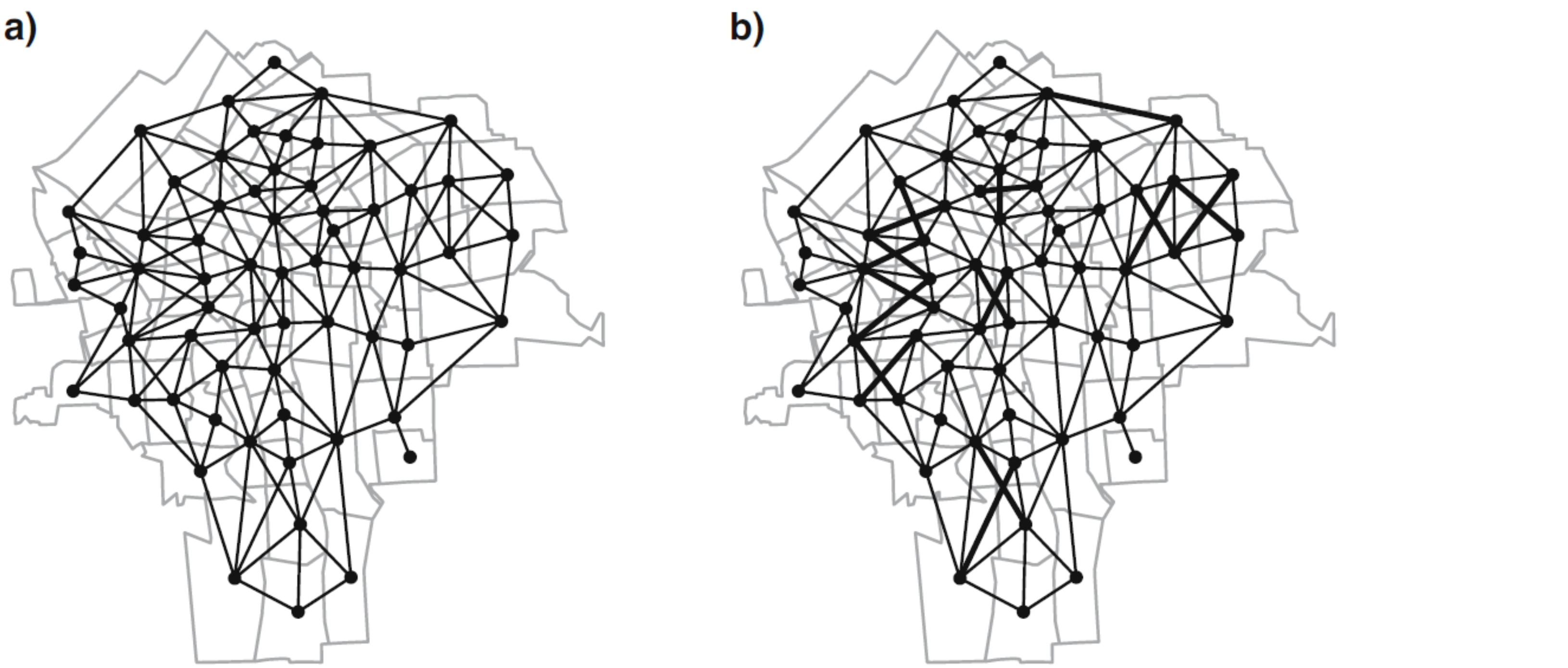


Fig. 9.3. (**a**) Queen-style census tract contiguities, Syracuse; (**b**) Rook-style contiguity differences shown as thicker lines

Bivand, R.S., Pebesma, E.J., Gómez-rubio, V., 2008. Applied Spatial Data Analysis with R. Springer, New York, NY

Other types of relationships

- Distance based
  - Based on Euclidean distance
  - K-nearest neighbors
  - Every neighbor inside of search distance
- Based on graph measures
  - Based on topological position based on centroids (or points on surface)
  - Delaunay Triangulation, Sphere of Influence, Gabriel graph, relative graph neighbors, minimal spanning tree
- Higher order neighborhood definitions possible
  - Neighbors of neighbors

K-nearest neighbors

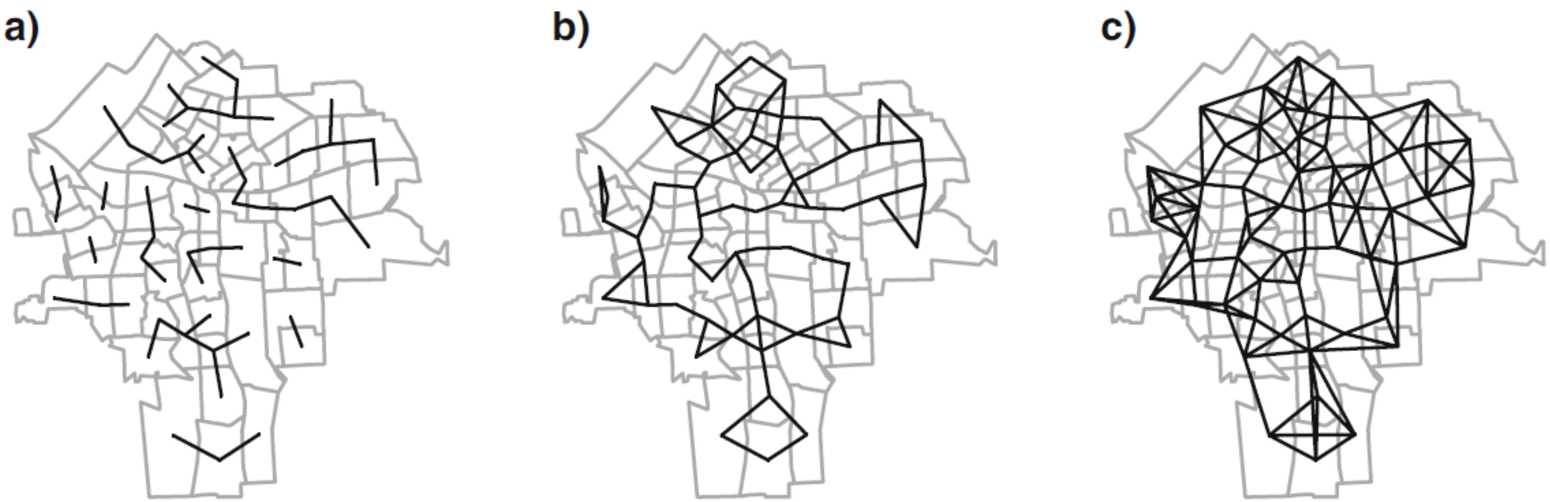


Fig. 9.5. (**a**) k = 1 neighbours; (**b**) k = 2 neighbours; (**c**) k = 4 neighbours

Bivand, R.S., Pebesma, E.J., Gómez-rubio, V., 2008. Applied Spatial Data Analysis with R. Springer, New York, NY

Distance threshold based neighbors

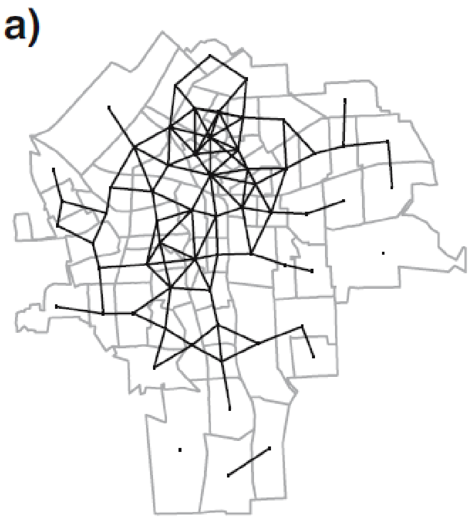


Fig. 9.6. (**a**) Neighbours within 1,158 m; (**b**) neighbours within 1,545 m; (**c**) neighbours within 2,317 m

Topological neighborhood definitions

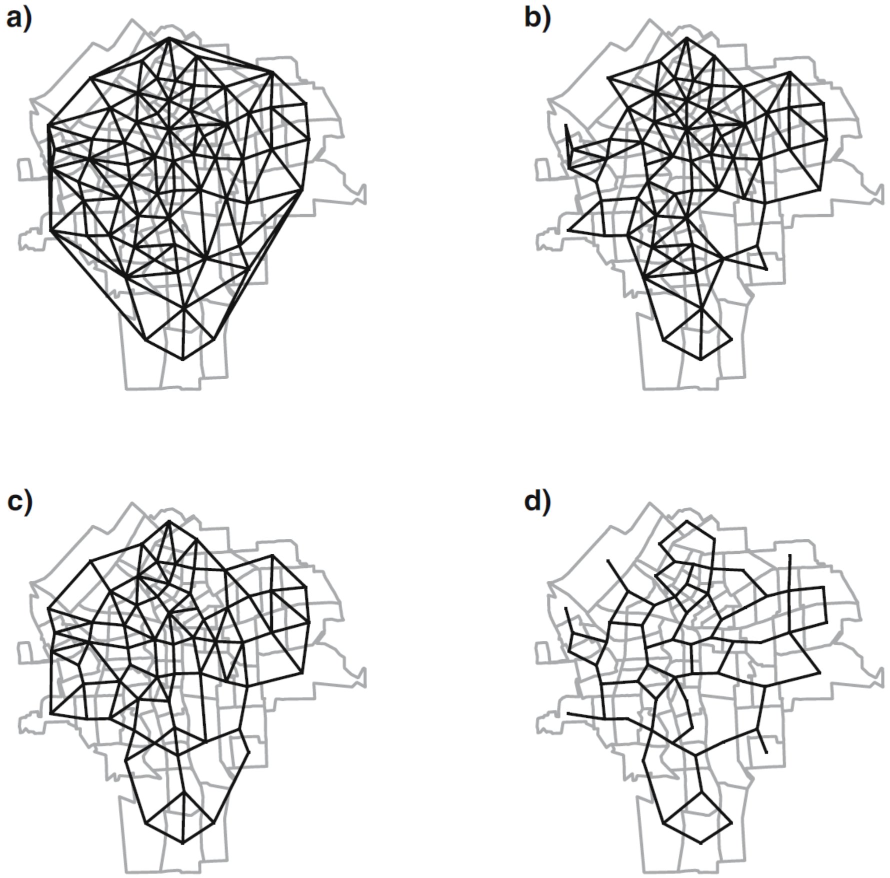
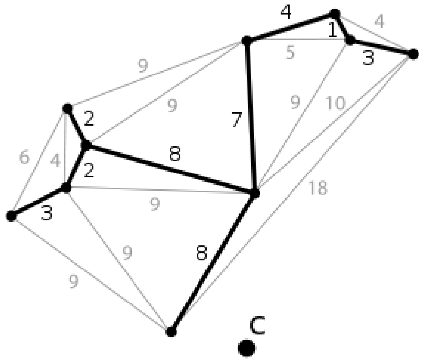


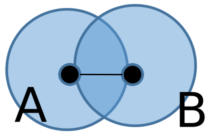
Fig. 9.4. (**a**) Delauney triangulation neighbours; (**b**) Sphere of influence neighbours; (**c**) Gabriel graph neighbours; (**d**) Relative graph neighbours

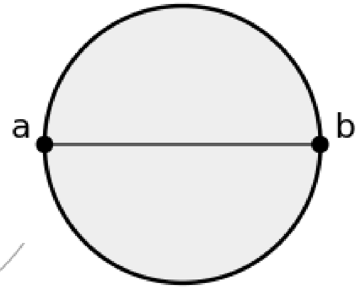
Bivand, R.S., Pebesma, E.J., Gómez-rubio, V., 2008. Applied Spatial Data Analysis with R. Springer, New York, NY

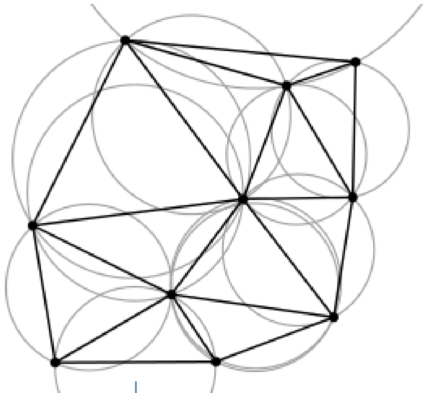
Topological neighborhood definitions

- Minimum spanning tree
  connects all nodes together while minimizing total edge length
- Relative neighborhood graph
  all nodes connected for those the lens formed by the radii of their circles contains no other points
- Gabriel graph
  all nodes connected if were is no other node inside a circle with their distance
- Delaunay triangulation
  all nodes connected for which the circumcise around ABC contains no other nodes



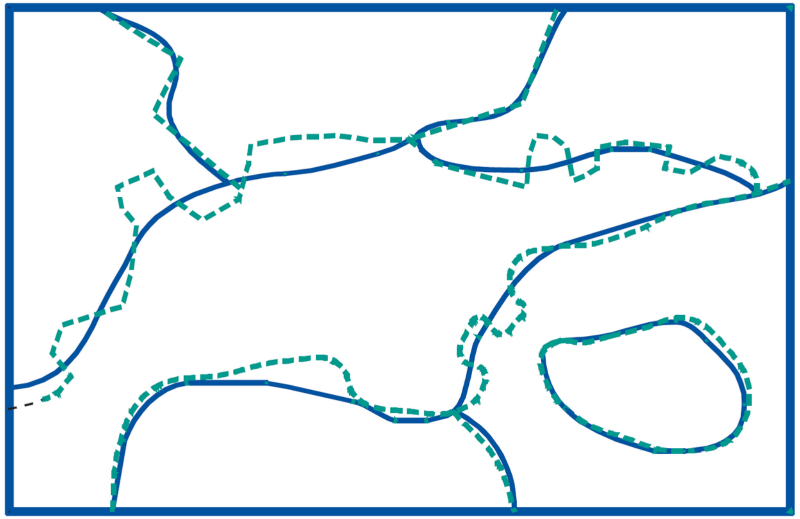






Selecting a neighborhood – things to consider

- GIS precision might lead to artifacts in neighborhoods for contiguity definition
  - Sliver polygons etc.
  - Fix it beforehand



Selecting a neighborhood – things to consider

- Is Euclidean distance important for the assumed process?
  - If you don't know you could investigate using different approaches
  - If distance is important we might assign weights based on distance. Use different approaches to investigate importance of distance

Selecting a neighborhood – things to consider

- Does the definition lead to objects without neighbors?
  - Artifacts or true islands?
  - Problematic for analysis
  - Exclude or set zero.policy
  - If zero policy is set to TRUE, weights vectors of zero length are inserted for regions without neighbor in the neighbors list
  - These will in turn generate lag values of zero
  - The spatially lagged value of x for the zero-neighbor region will then be zero, which may (or may not) be a sensible choice

Assigning weights

- How to deal with uneven neighborhood distributions?
- Binary coding *B* – neighbor (1) or not (0)
  - Objects with many neighbors get more weight
- **Row standardized coding** *W* : weight (0 or 1) divided by the number of neighbors, i.e. weights sum to unity for each object (sums over all links to n)
  - Mostly used in practice, assumed for some approaches, the effect of the neighbors is expressed as the weighted sum
- Globally standardized coding *C*: weight (1 or 0) divided by sum of all weights, i.e. sum to n for all objects
- *U*: equal to *C* divided by the number of neighbors (sums over all links to unity)
- *S*: variance-stabilizing coding scheme proposed by Tiefelsdorf et al. 1999, p. 167-168 (sums over all links to n). Between *W* and *C*

Selecting a neighborhood – things to consider

- Neighborhood frequency distribution even or skewed?
  - Contiguity -> typically relatively equal distribution
  - Distance based -> skewed
  - K-nearest -> equally distributed
- K-nearest does **not** lead to a symmetric relationship!
- Some analysis require symmetric relationships
  - It is possible to make a nb relationship symmetric by adding neighbors (*make.sym.nb*)
- It is also possible to use set operations on nbs (intersect, union, difference, complement) or to manually modify nbs

Selecting a weighting scheme – things to consider

- Beside coding style *U* all coding styles sum to n over all links → estimated spatial auto-correlation should be comparable
- In addition, weights can be inversely weighted by distance prior to standardization
- *S* and *W* might lead to non-symmetric weight matrix

Spatial exploratory analysis

What are we looking for?

- DATA = SMOOTH + ROUGH (Tukey, 1975)
- SPATIAL DATA = SPATIAL SMOOTH + SPATIAL ROUGH
-

What are we looking for?

- Spatial smooth
  - Presence of spatial trend?
  - Spatial heterogeneity – is variation in data values as smooth as implied by some trend?
  - Global spatial dependence – are high/low values close to other high/low values, anywhere on the map?
  - Spatial heterogeneity – localized patterns of dependence? Hot-/coldspots?

What are we looking for?

- Spatial rough
  - Presence of outliers?
    - In addition to distributional outliers: are there spatial outliers (which might not be distributional outliers)
    - i.e. are there some data points special with respect to their neighborhood?

Moran’s I

- Measures global spatial auto-correlation (for all observations)
- I > 0 positive auto-correlation, clustered
- I < 0 negative auto-correlation, dispersed
- Expected value under absence of spatial auto-correlation E(I) = $-n(n-1)^{-1}$

$$I = \frac{1}{\sum_{i=1}^{n} w_{i,j}} \frac{\sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} w_{i,j}(x_i - \bar{x})(x_j - \bar{x})}{\frac{1}{n}\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

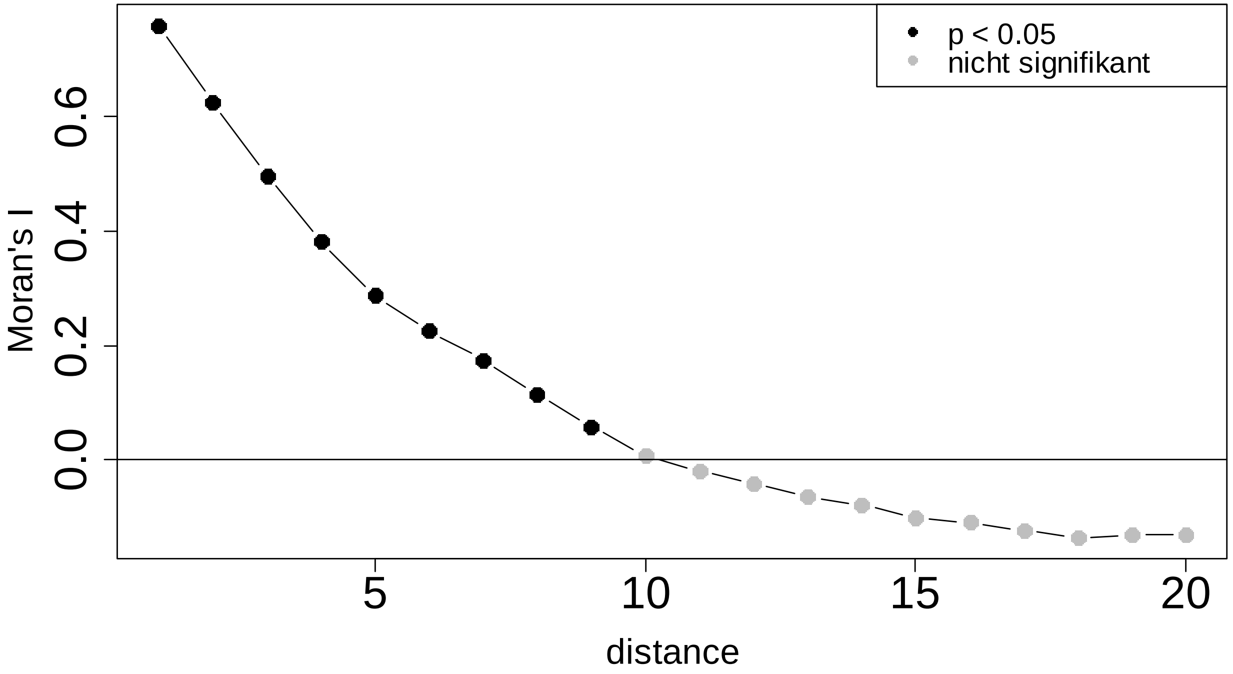
  - $w_{i,j}$ – weights for observation I and j from the weight matrix W
  - x_i – value of observation *i*

Moran‘s I

- Not bound to the interval [-1,1]
  - Might shrink but typically expand
  - Interval defined by the largest and second largest eigenvalue of the weight Matrix W
  - Often the interval is more in the range -0.5 to 1.15
  - The expected value for no autocorrelation is not zero but E(I) = -1/(n-1)
- For regression residuals a modified test statistic is used
- For rates the empirical Bayes index modification is used

Correlogram

- Plot of Moran‘s I against distance classes
  - Significance testing via permutation under H_0

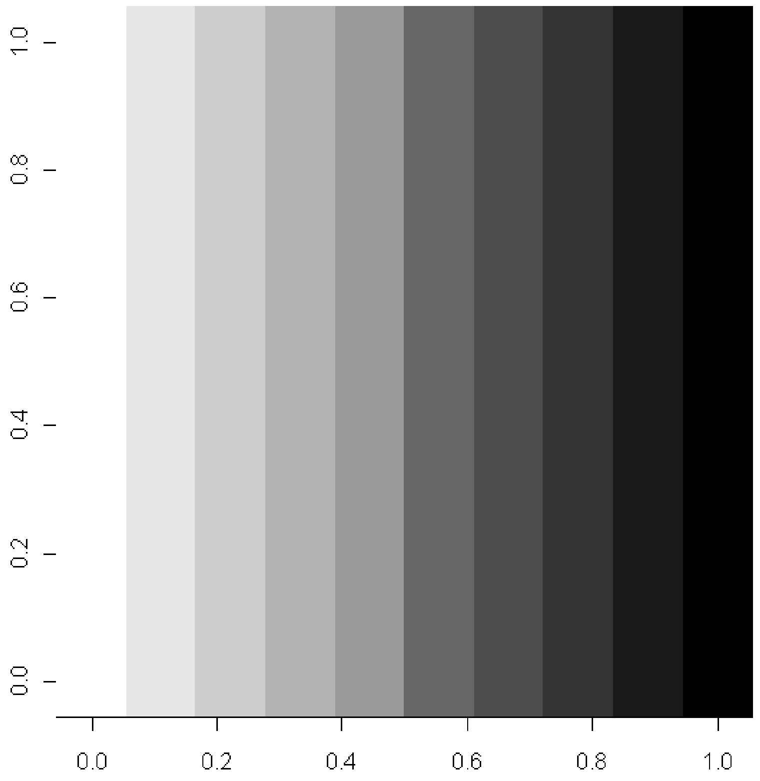


$$I(d) = \left(\frac{1}{W(d)} \right) \frac{\sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{\substack{j=1 \\ j \neq 1}}^{n} w_{i,j}(d)(x_i - \bar{x})(x_j - \bar{x})}{\frac{1}{n} \sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2}}$$

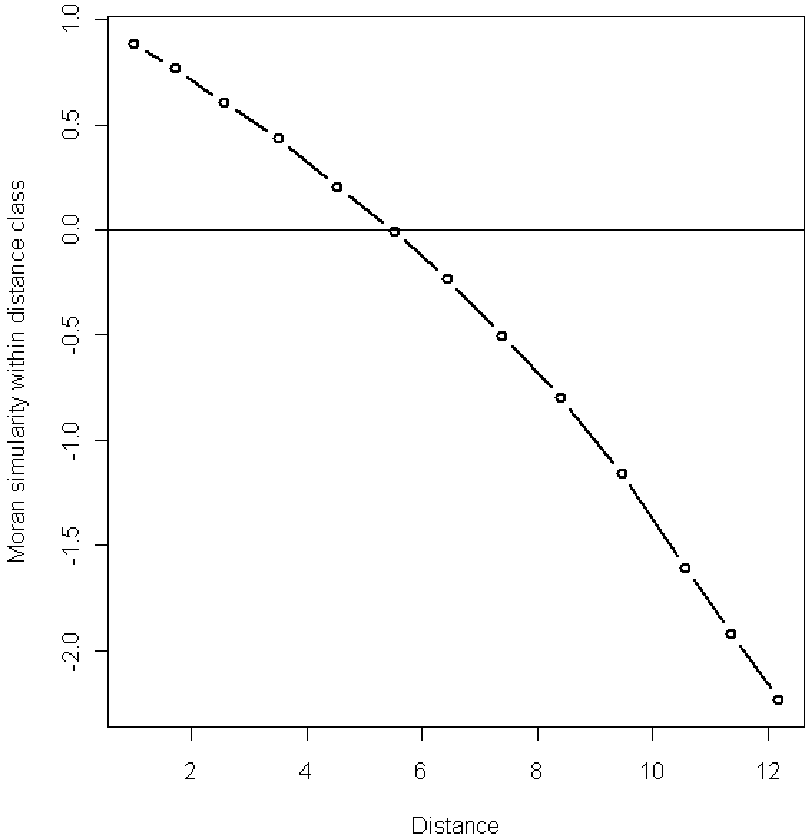
D – Distance class
$W_{i,j}$ – neighbor or not
W(d) – sum over all $w_{i,j}$

Examples - Gradient

Value raster

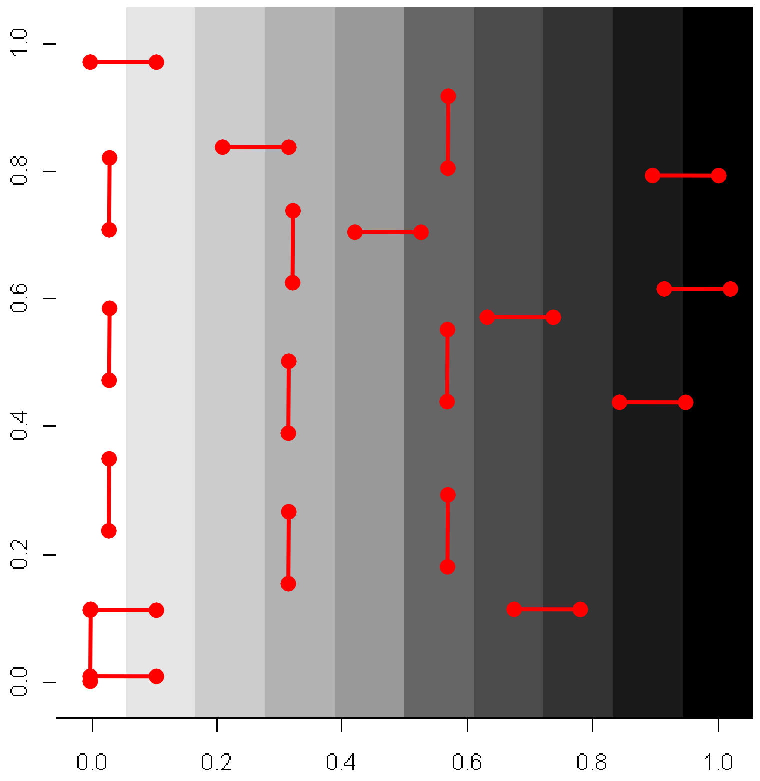


Correlogram that shows how spatial autocorrelation changes with distance

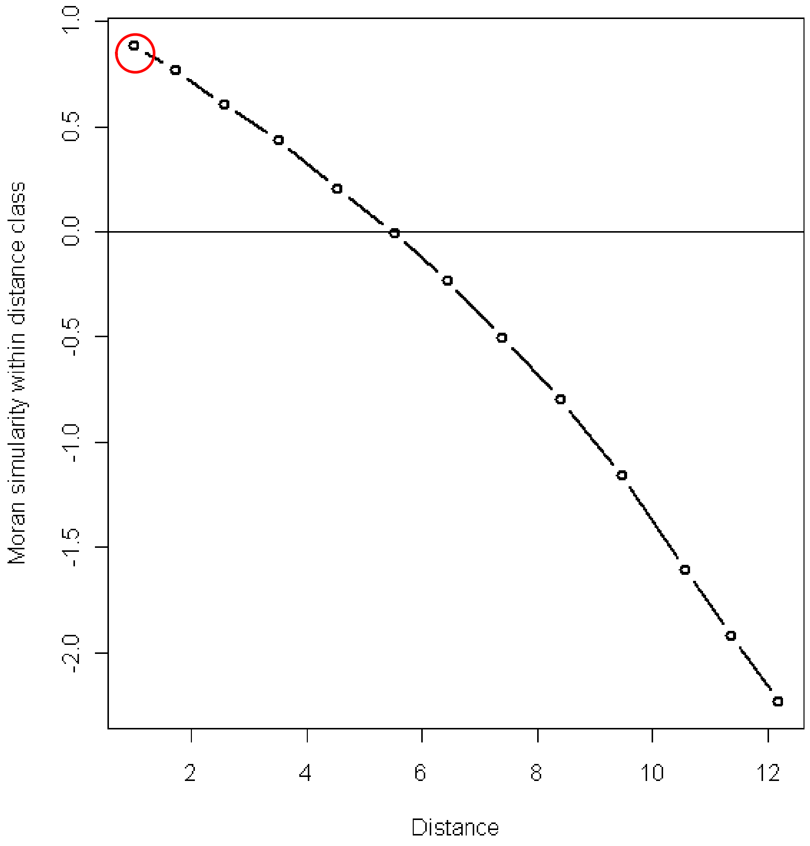


Example - Gradient

Some pairs of distance class 1



Moran‘s I for all pairs of distance class 1



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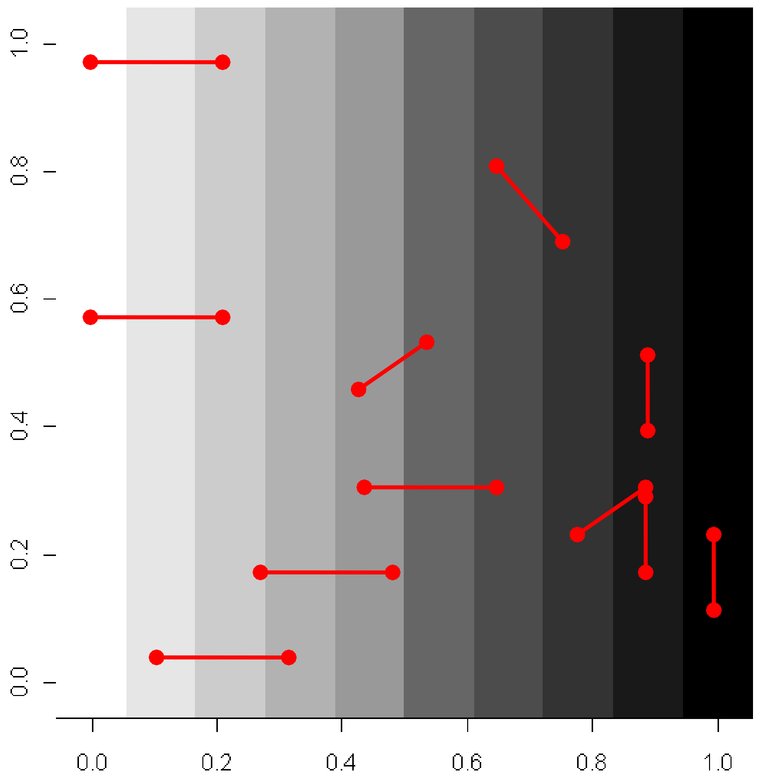
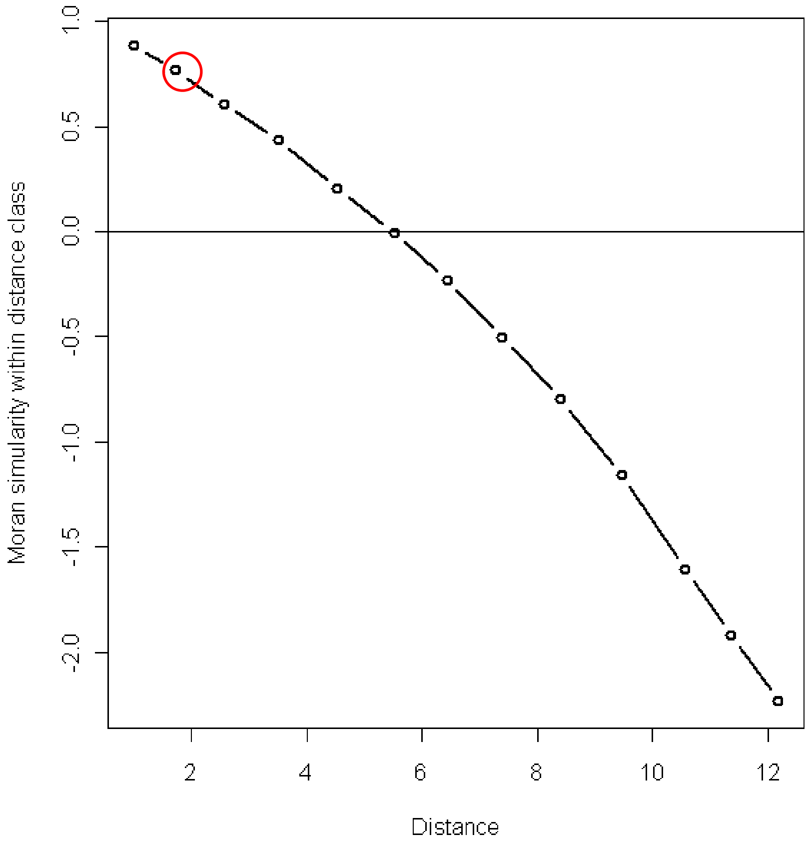
Example - Gradient

Some pairs of distance class 2

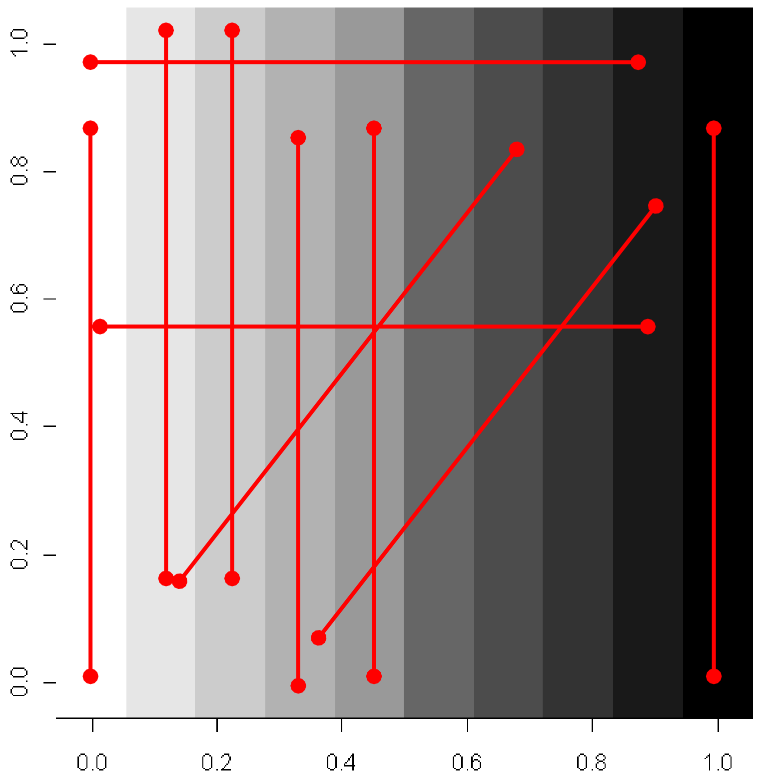


Moran‘s I for all pairs of distance class 2

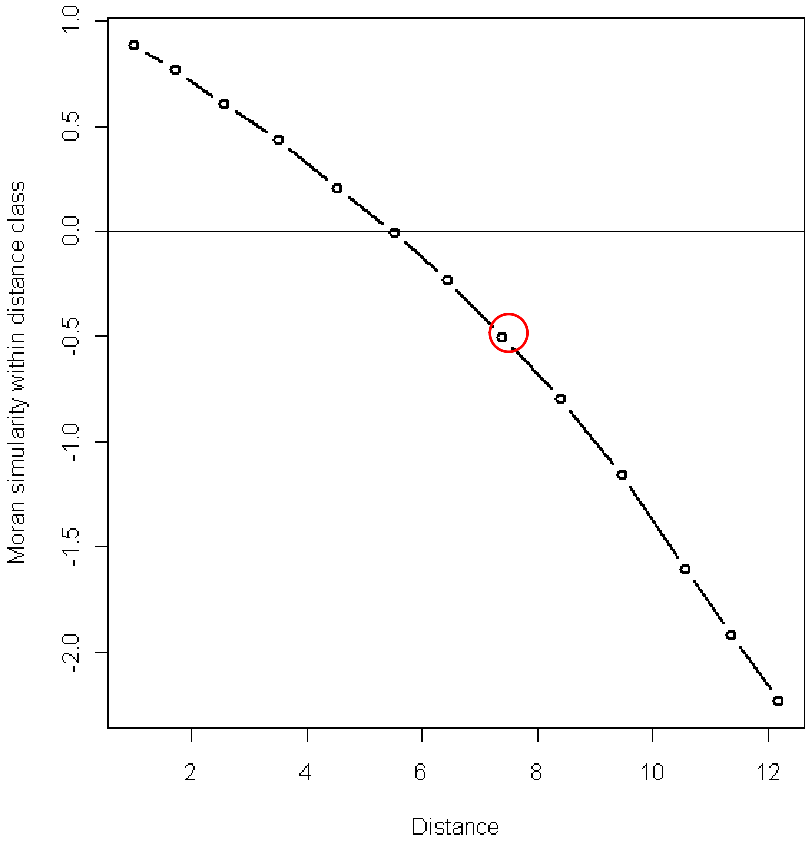


Example - Gradient

Some pairs of distance class 8

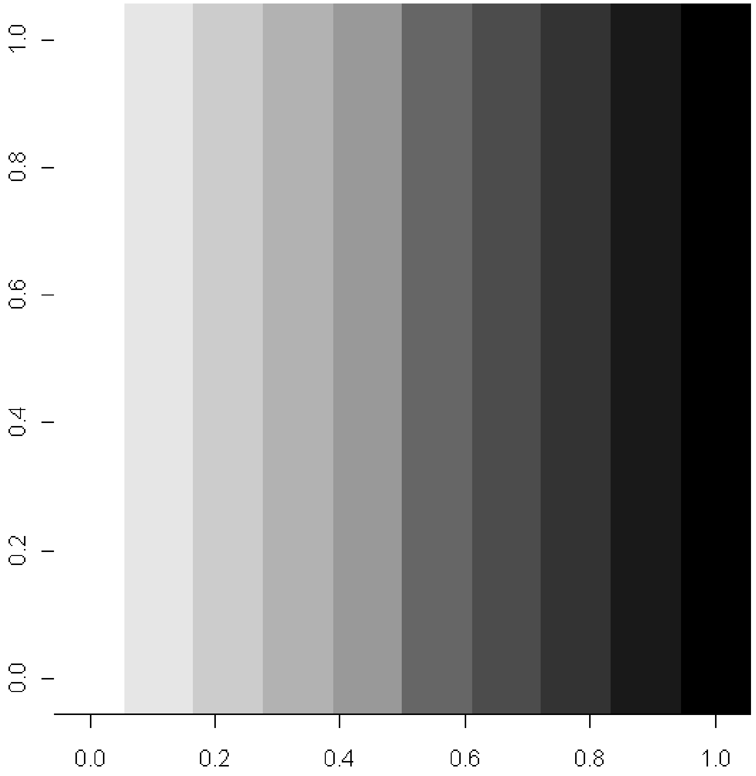


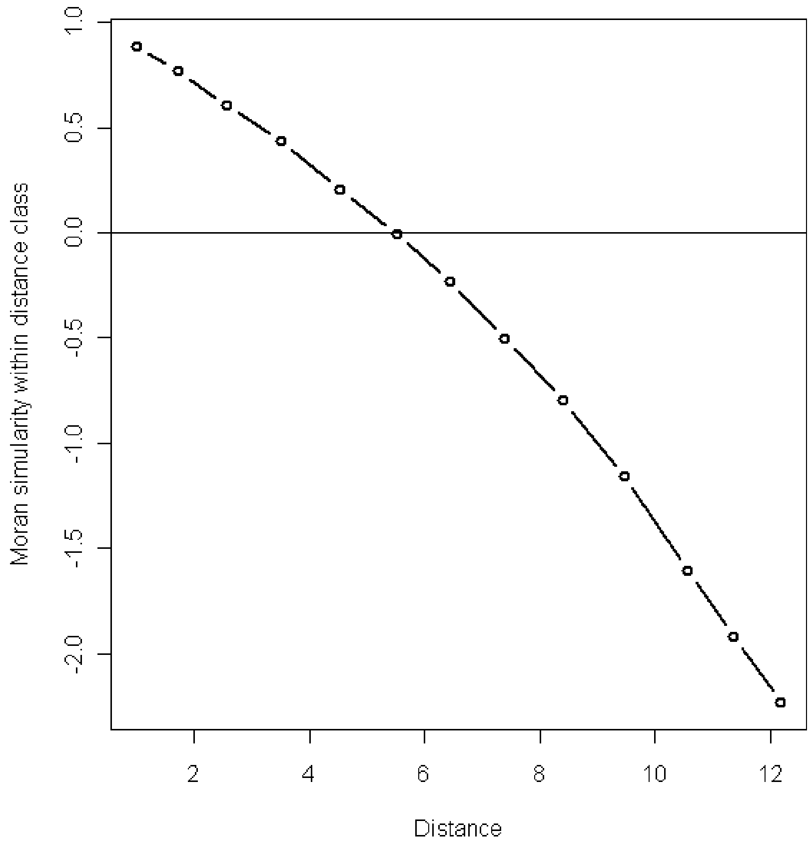
Moran‘s I for all pairs of distance class 8



Example - Gradient

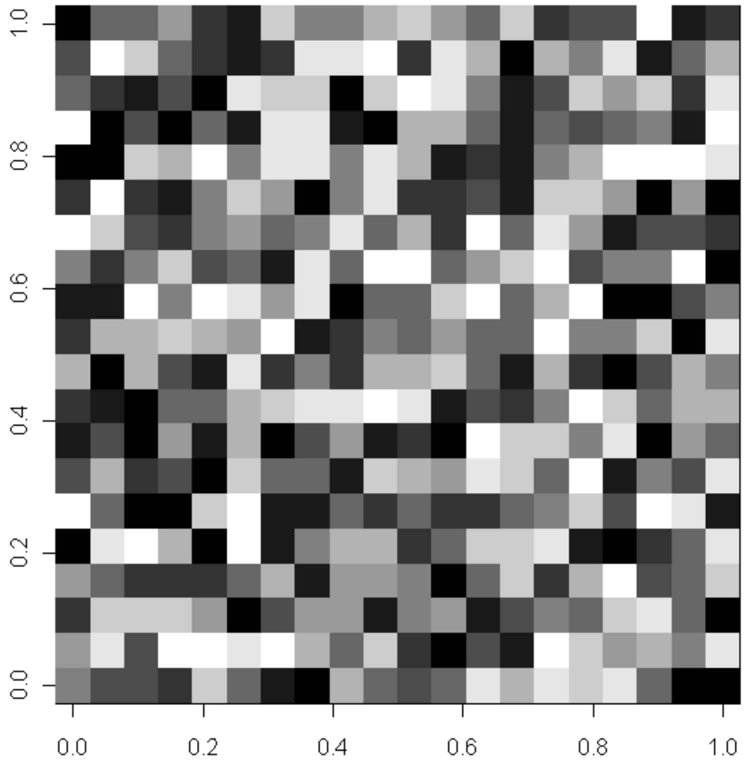
Close observations are very similar. Effect diminishes with increasing distance and turns negative at a distance of 6

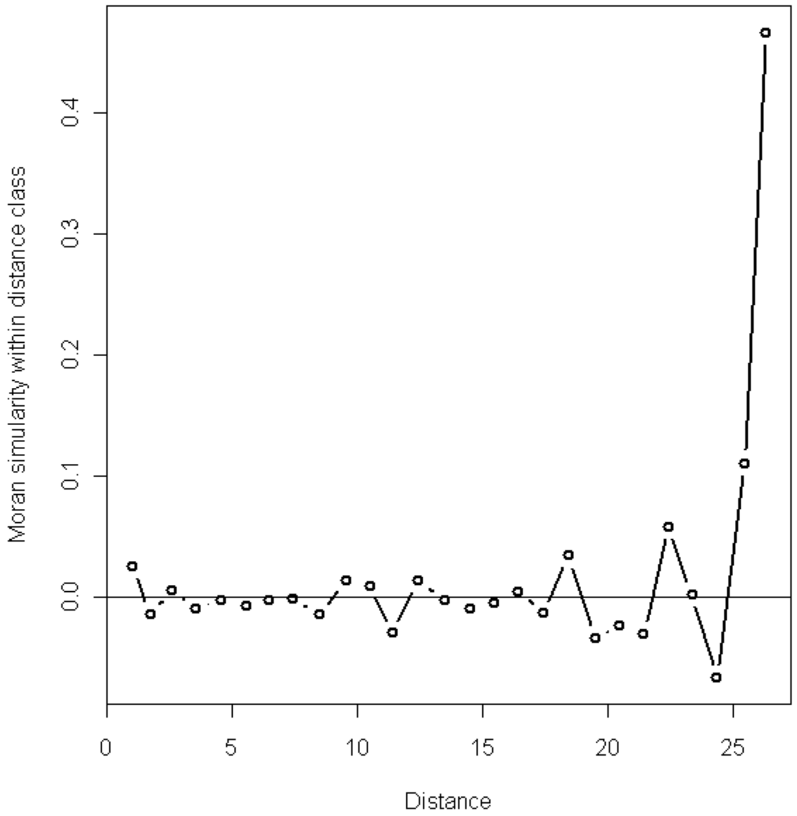




Random pattern

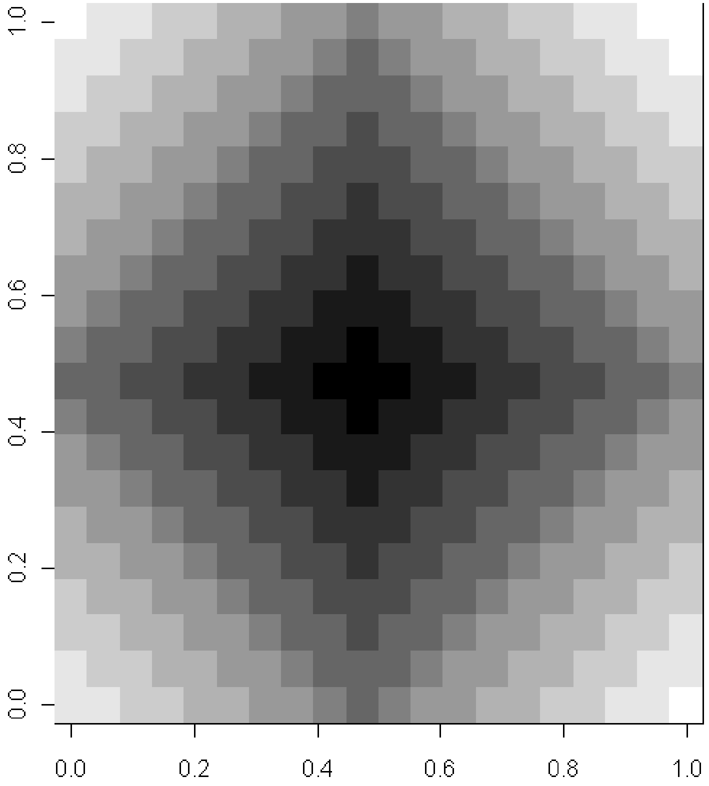
Peak at distance class 26 is an artefact – based on only a few observation. Therefore, large distance classes should not be plotted!

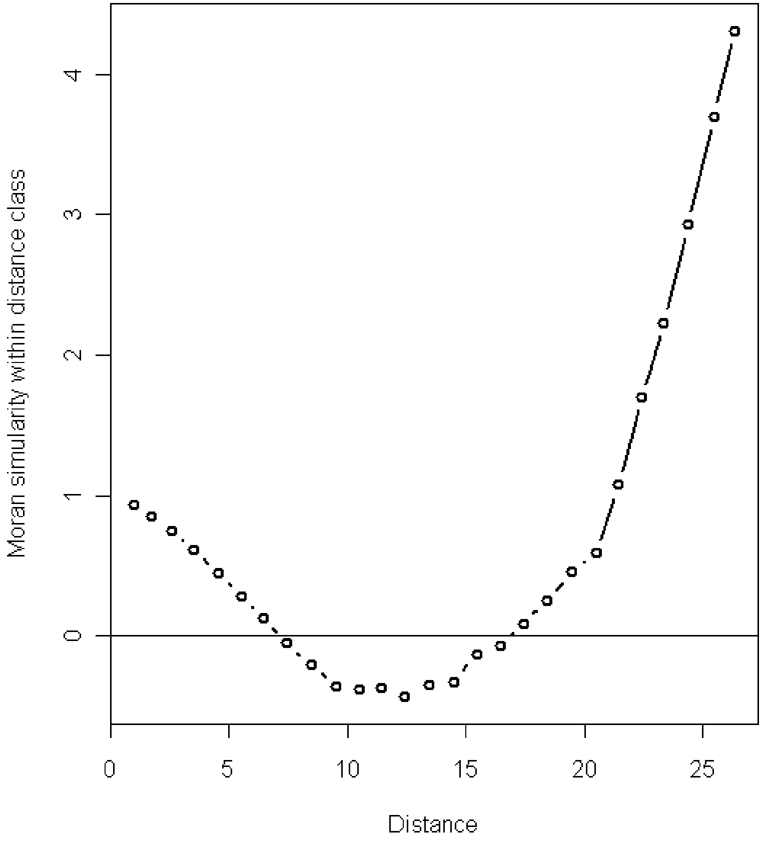




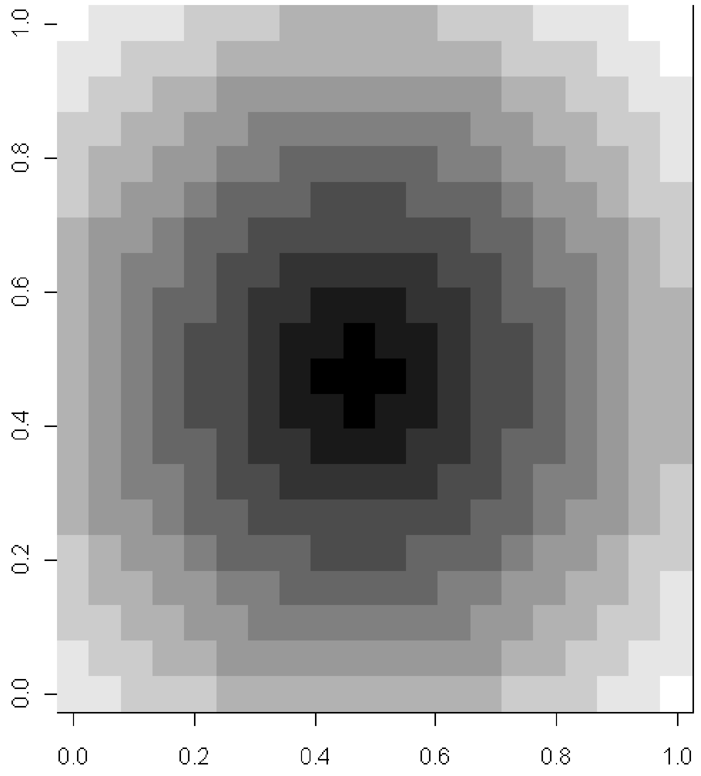
Concentric pattern

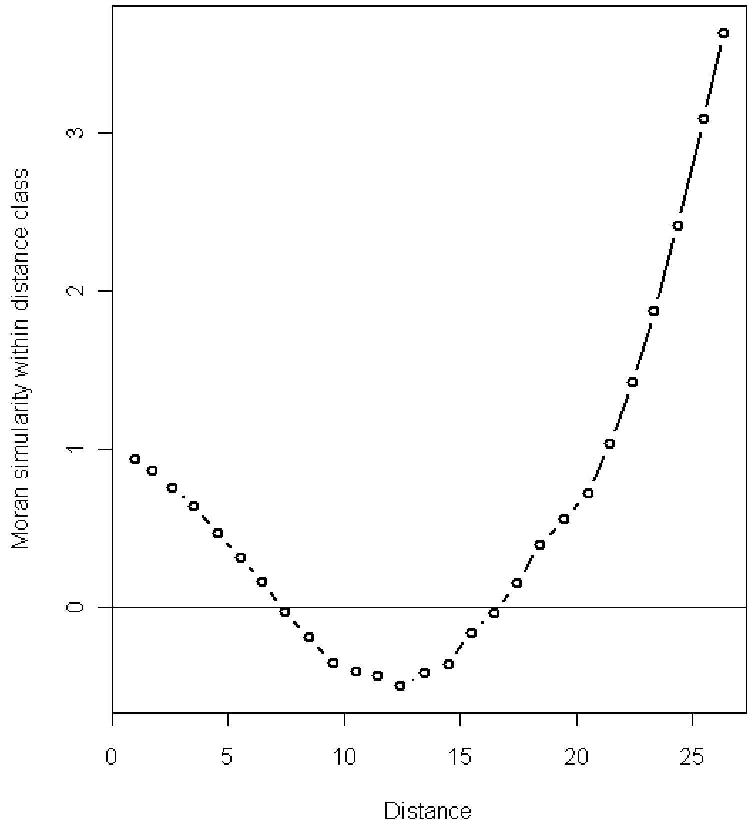
Pattern for distance classes larger than 15 are not reliable here



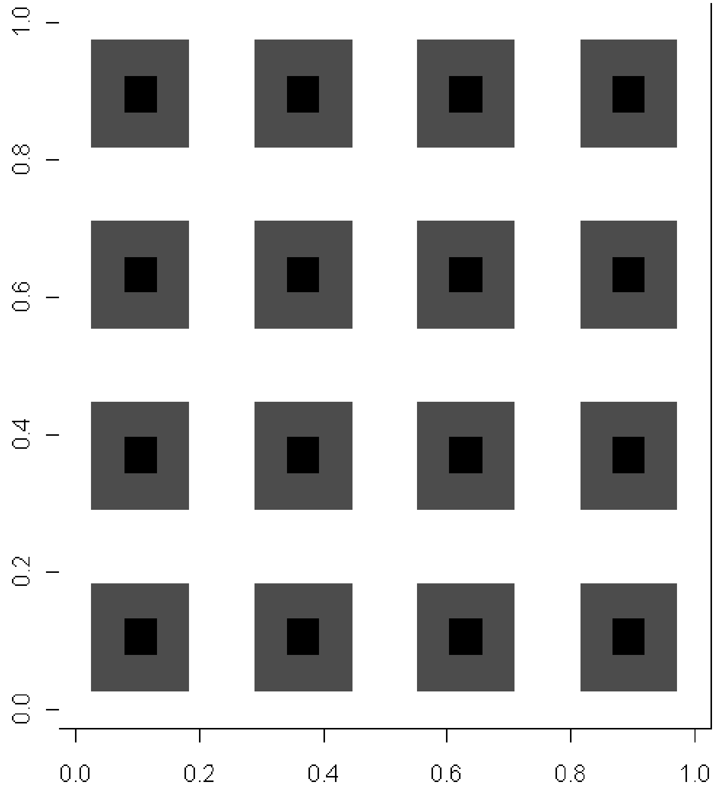


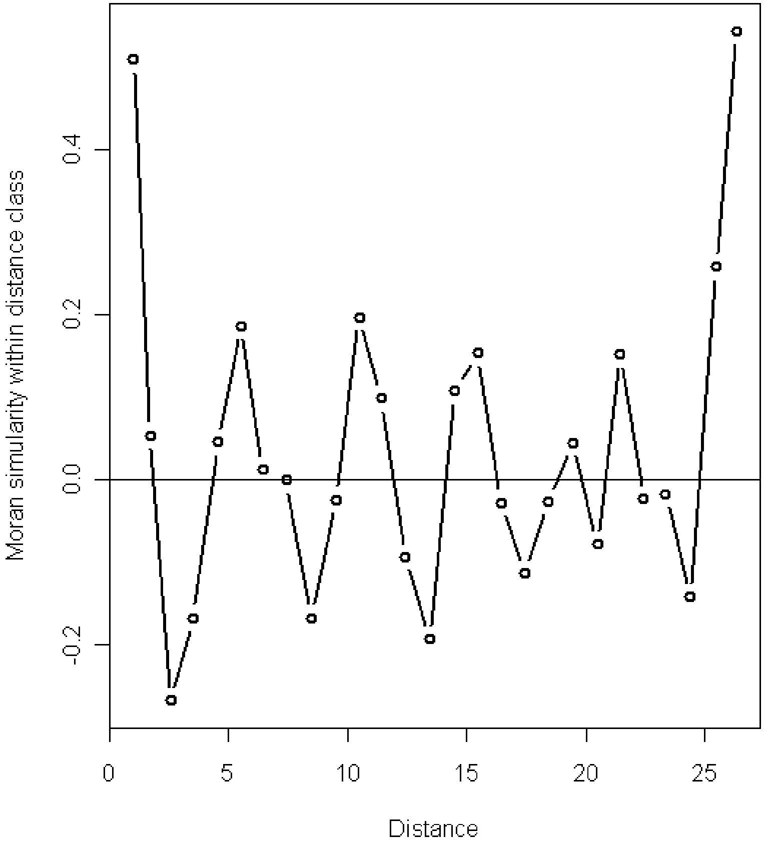
Concentric pattern



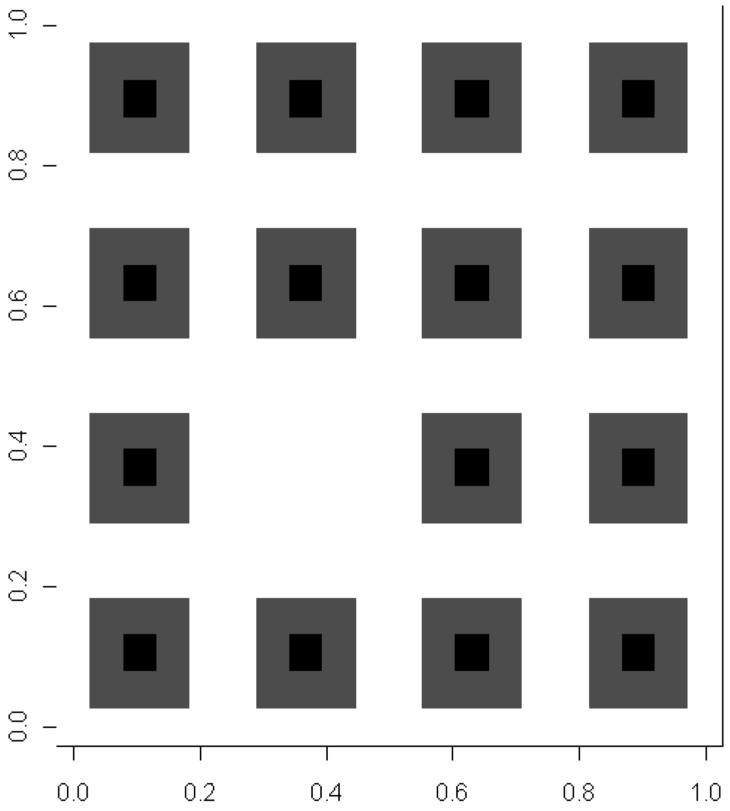


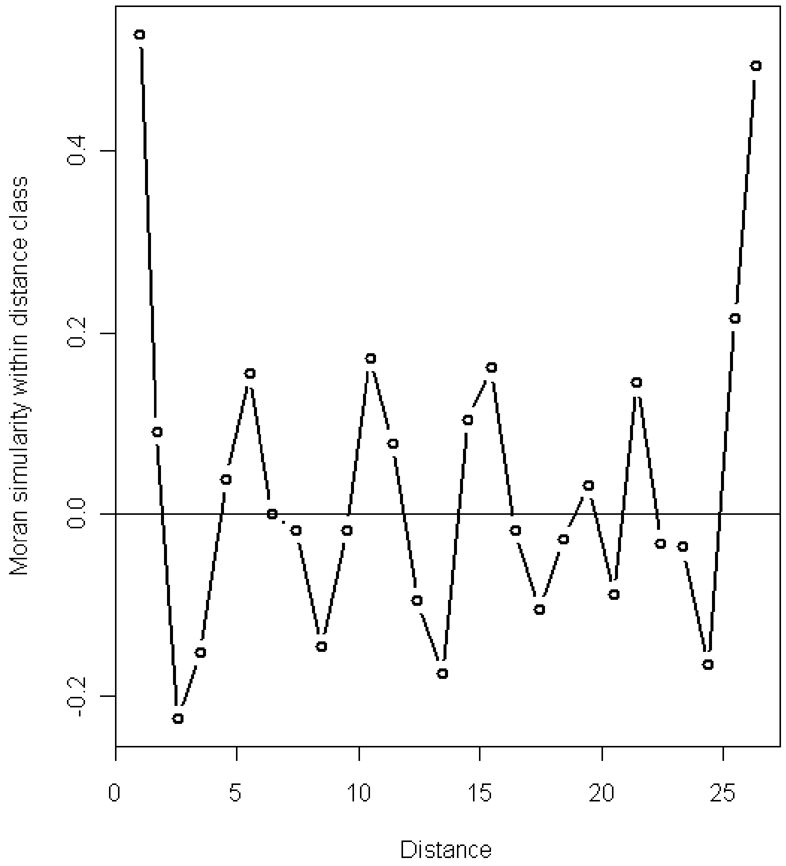
Patchy pattern



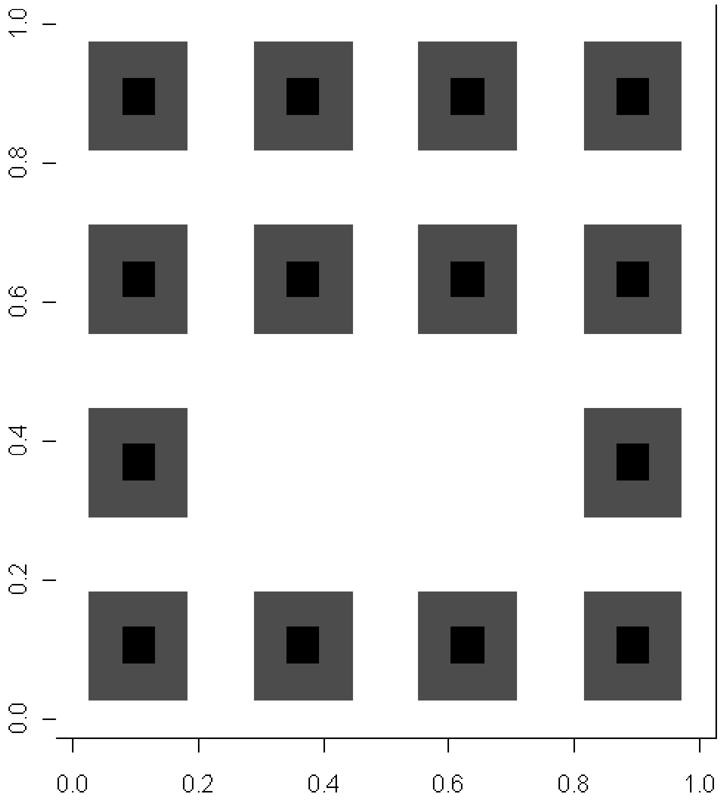


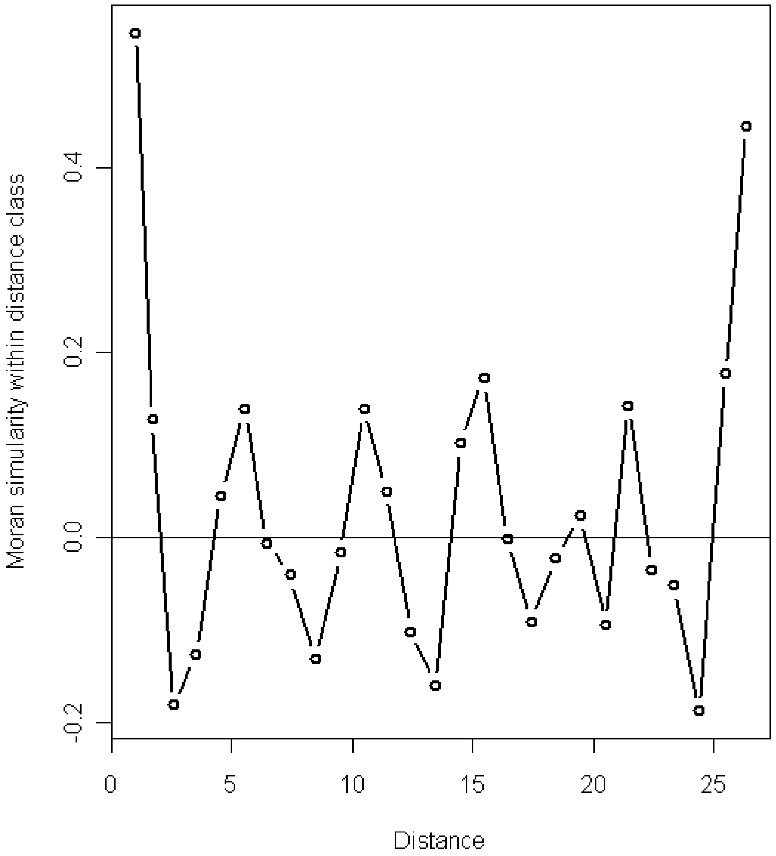
Patchy pattern with holes



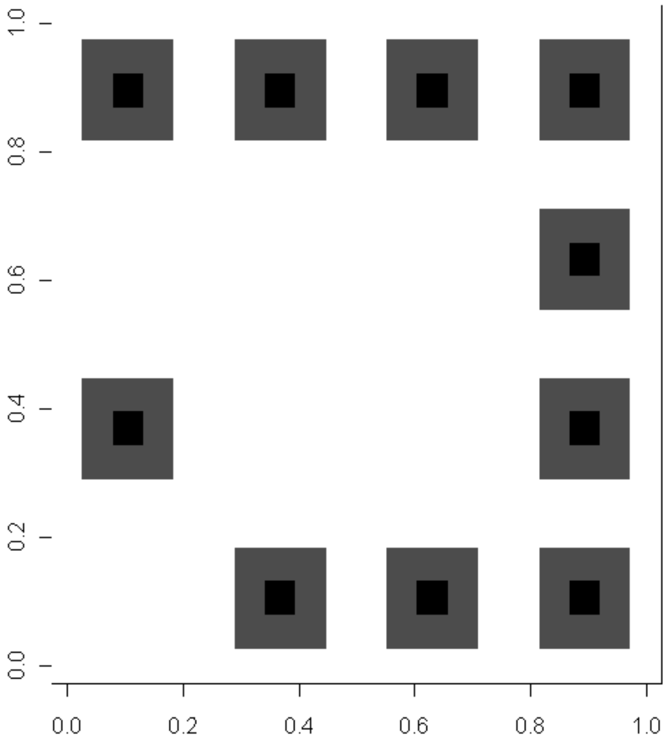


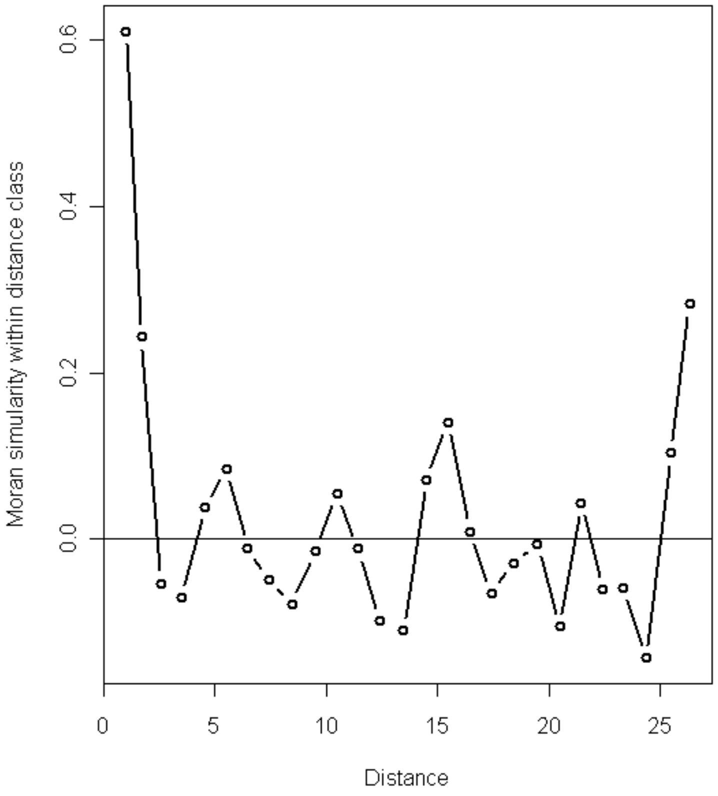
Patchy pattern with holes



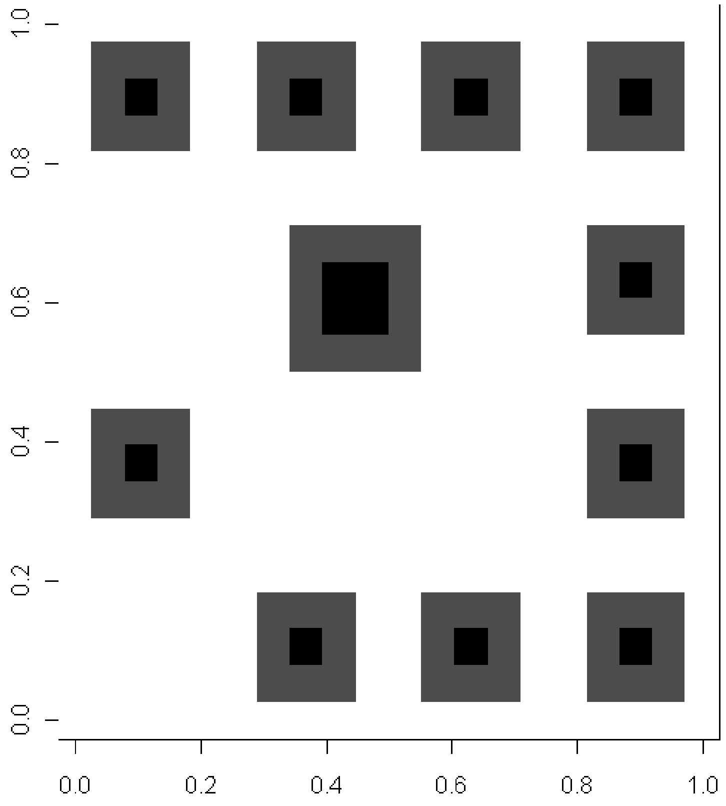


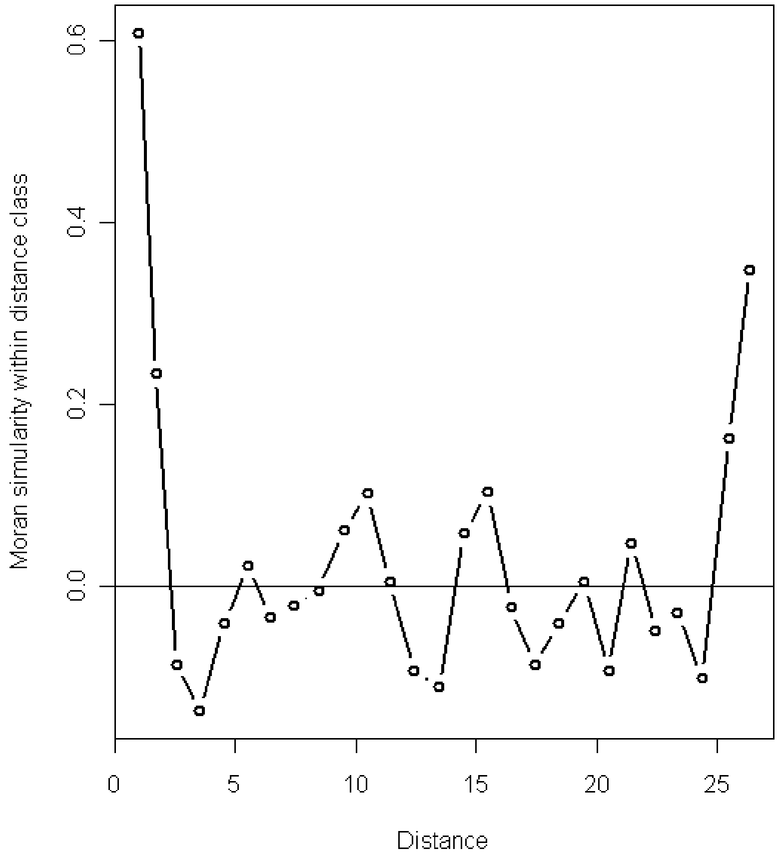
Patchy pattern with holes



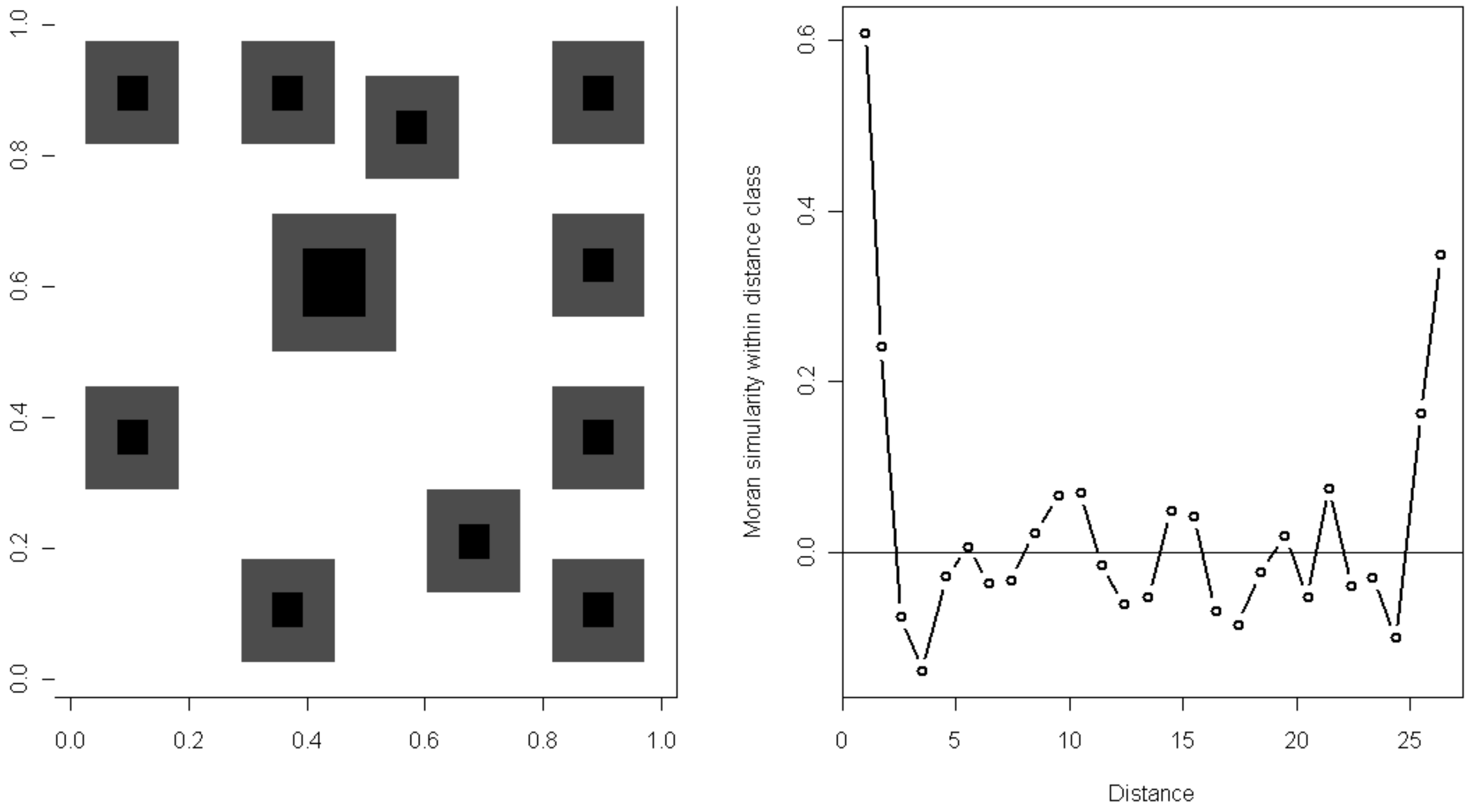


Patchy pattern with holes, different size of patches

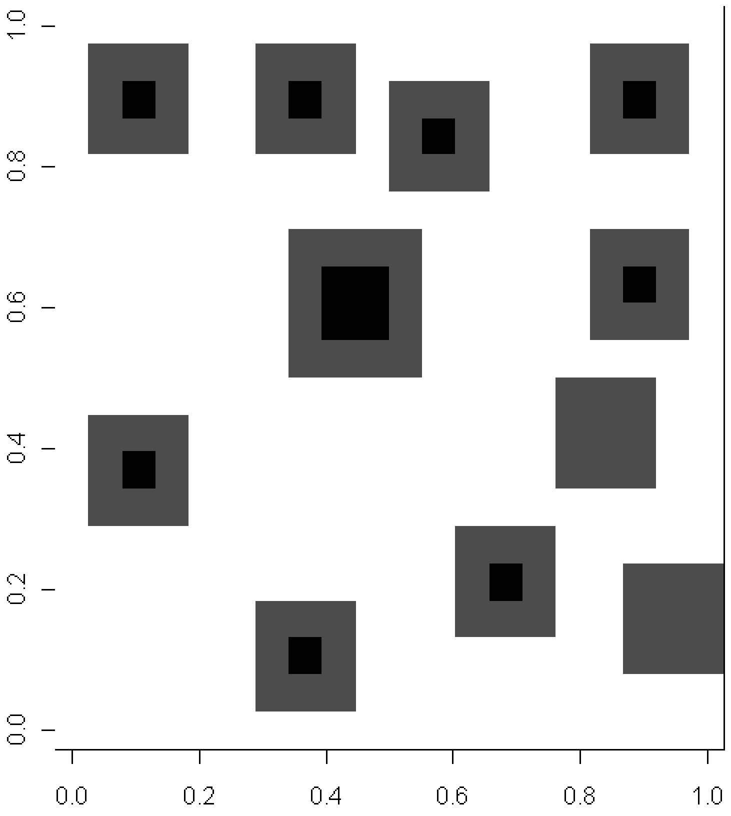


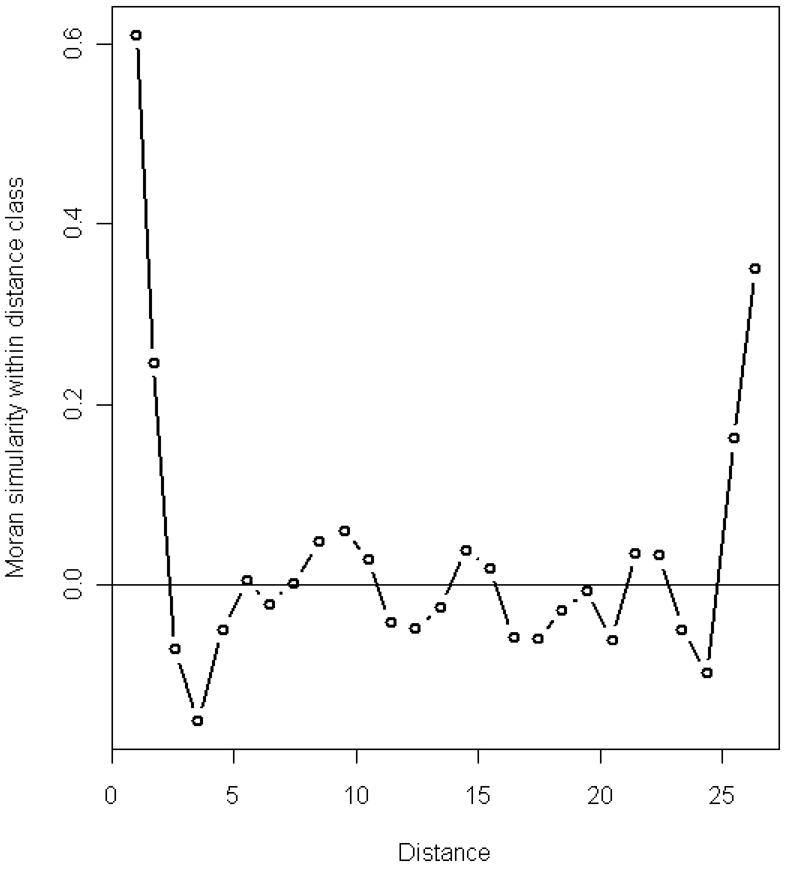


Patchy pattern with holes, different size of patches

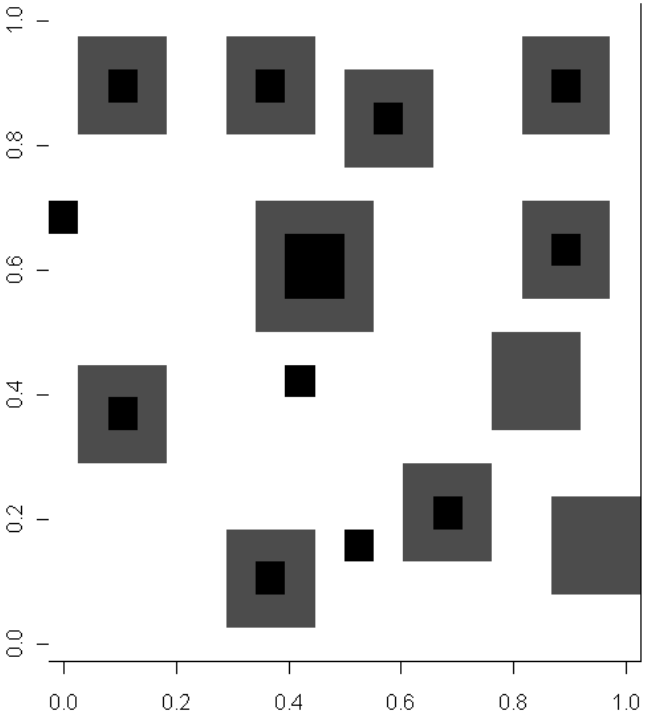


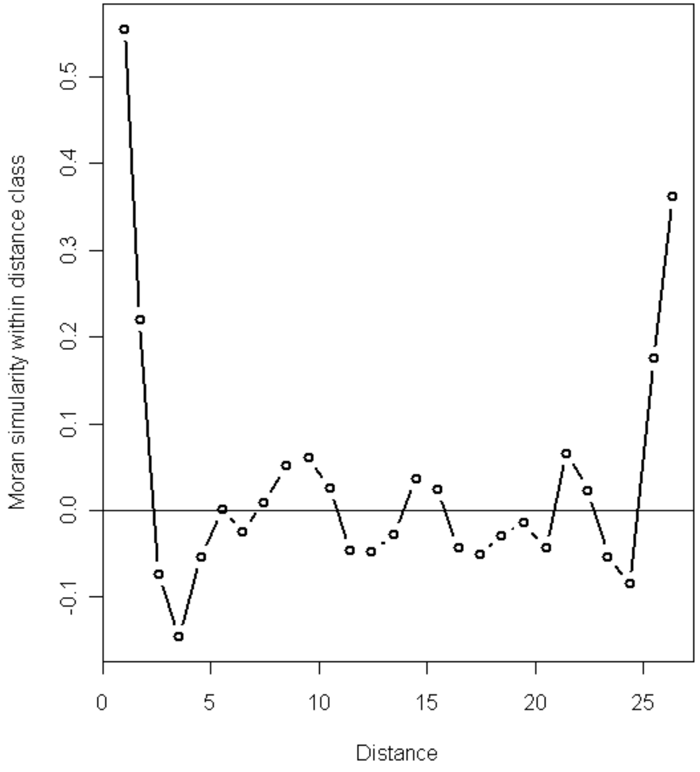
Patchy pattern with holes, different size of patches





Patchy pattern with holes, different size of patches

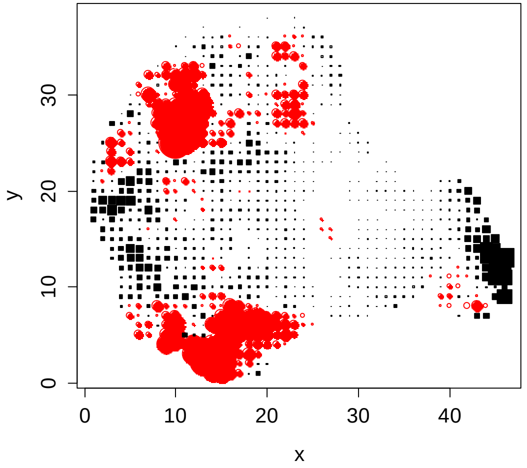




LISA - local indicator of spatial association

- E.g. local Moran‘s I
- Spatial auto-correlation based on a focal function
- Takes instationarity into account

$$I_i = n(x_i - \bar{x}) \frac{\sum_{\substack{i \neq j \\ i=1}}^{n} w_{i,j}(x_i - \bar{x})}{\sum_{i=1}^{n} (x_i - \bar{x})}$$



Local Moran‘s I

- Calculates Moran’s for the neighborhood of a point and relates it to the mean over all data

Mean for all observations

$$I_i = n(x_i - \bar{x}) \frac{\sum_{\substack{i=1 \\ i \neq j}}^{n} w_{i,j}(x_i - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})}$$

$$I = \frac{1}{n}\sum_{i=1}^{n} I_i$$

No double sum as in global Moran‘s I
Only sum over the observations in the neighborhood

Critic local Moran‘s I

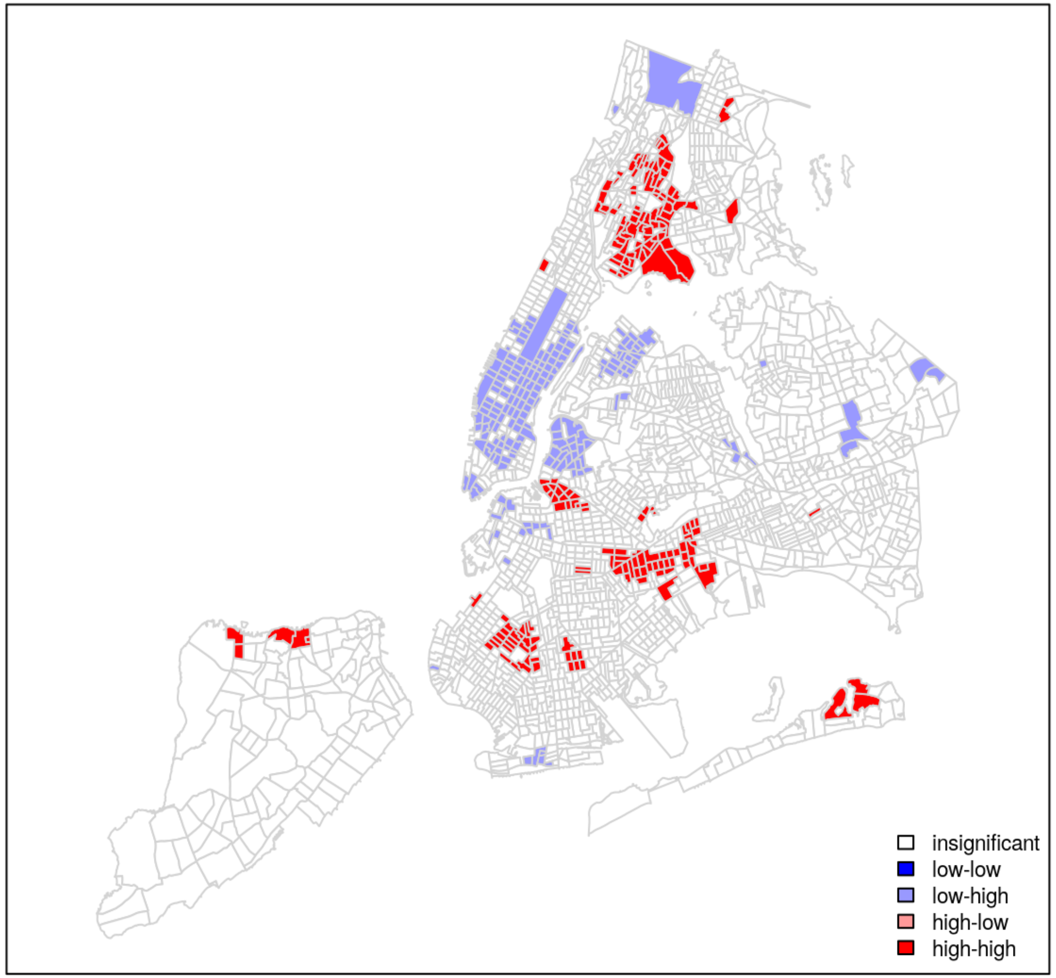
- Direction of change is not available
  - High values at location i, surrounded by high values in neighborhood -> positive value
  - Low values at location i, surrounded by low values in neighborhood -> positive value
  - High values at location i, surrounded by low values in neighborhood -> negative value
  - Low values at location i, surrounded by high values in neighborhood -> negative value

Local Moran‘s I

- Therefore often the value of the observation is reported together with the average value in the neighborhood
- High – high – cluster of high values
- Low – low – low values (low relative to the global mean) in a neighborhood of high values
- High –low – high value (compared to the mean) in a neighborhood of negative spatial autocorrelation, i.e. high local outlier
- Low - high – low value (compared to the mean) in a neighborhood of positive spatial autocorrelation, i.e. a cluster of low values

Local Moran‘s I

LISA on percent inhabitans younger than 21, SOI nb



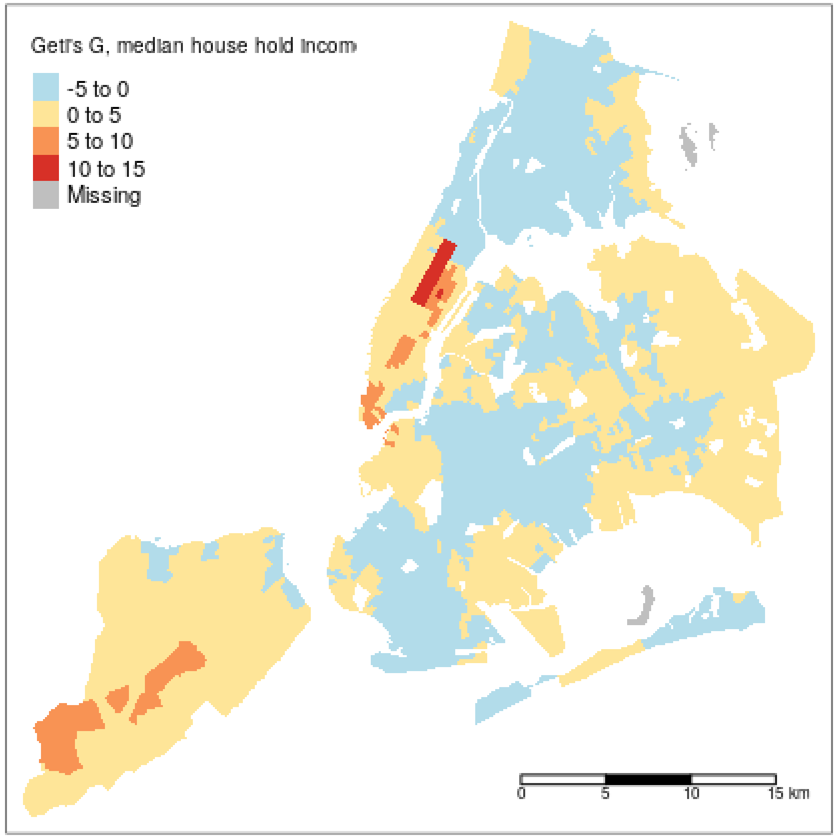
Getis G and G*

- Relation between the local mean at location I to the global mean (local moving average, hotspot/coldspot)
- Sign indicates direction of deviation
- G_i^* and G_i differ by that G_i^* does include the point itself

$$G_i = \frac{\sum_{\substack{j=1 \\ j \neq i}}^{n} w_{ij} x_i x_j}{\sum_{\substack{j=1 \\ j \neq i}}^{n} x_i x_j}$$

$$G_i^{star} = \frac{\sum_{j=1}^{n} w_{ij} x_i x_j}{\sum_{j=1}^{n} x_i x_j}$$

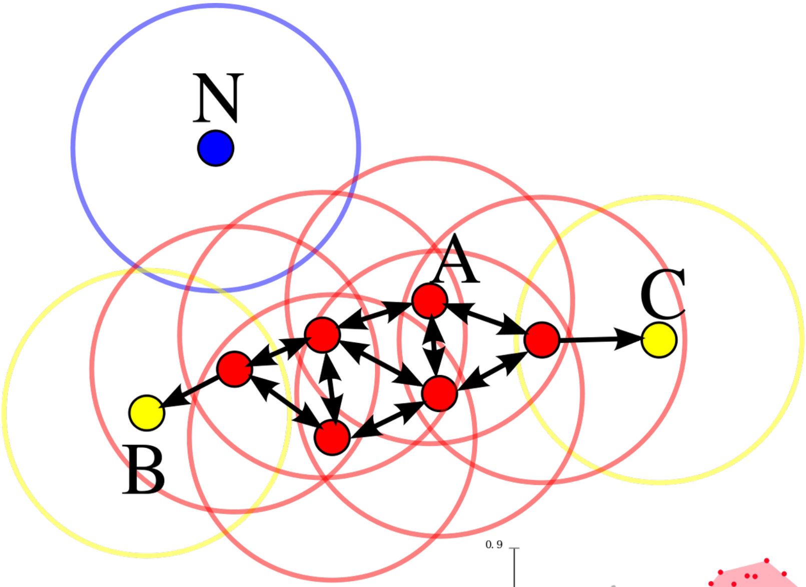
Getis G and G*

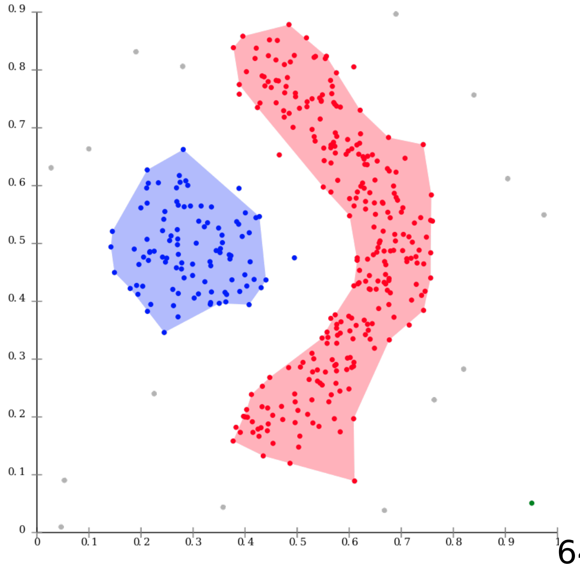

Getis G, median house hold incom
-5 to 0
0 to 5
5 to 10
10 to 15
Missing
0
5
10
15 km

Spatial cluster identification

dbScan

- Aims to identify spatially clustered points
- A cluster is defined as the subset of points that can be reached from all points in the cluster by a distance less than a threshold
  - In addition to core points there are points at the edges that can be reached from the cluster
  - Core points: at least minP points in distance
- Point outside clusters are considered noise points





Adjusting for spatial autocorrelation in regression models

How to deal with it?

- Incorporate it in additional covariates
  - Capture the spatial configuration in additional covariates and add them to the model
  - **Spatial eigenvector mapping**
  - Auto-covariate Regression
  - Wavelet analysis
- Adjust the error term
  - Fit a variance-covarinace matrix based on the non-independence of spatial observations
  - GLS and GLMM – error structure needs to be assumed
  - Simultaneous autoregressive error models (SAR) and conditional autoregressive models (CAR)
  - Generalized estimating equations (GEE) split the data into smaller clusters before modelling the variance-covariance relationship

How to deal with it? (2)

- Adjustments of test statistics
  - Dutilleul’s modified t-test, {SpatialPack}
  - CRH-correction for correlations {SpatialPack}
- Lagged response modells
  - The response depends on the response of the neighboring units
- Lagged predictor effects (SLX model)
  - Spill-over effect from in neighboring units
  - Artifacts due to the artificial spatial discretization
    - Are administrative units well suited for health data?
    - MAUP

Properties of some approaches

method	residuals	computational intensity
GAM	normal, Poisson, binomial	low
autogressive models (SAR/CAR)	normal	medium-high
GLS	normal	medium-high
GEE	normal, Poisson, binomial	low
autocovariate regression	normal, Poisson, binomial	low
spatial GLMM	normal, Poisson, binomial	very high
Spatial Eigenvector Mapping	normal, Poisson, binomial	very high

- Wavelet and GEE: flexible with distributional assumption, no categorical variables possible

Spatial autocorrelation in residuals
Spatial error model

- Incorporates spatial effects through error term

- Where:

$$y = x\beta + \varepsilon$$
$$\varepsilon = \lambda W \varepsilon + \xi$$

ε : vector of error terms, spatially weighted using weights matrix W

λ : spatial error coefficient

ξ : vector of uncorrelated error terms

- If there is no spatial correlation between the errors, then = 0

Spatial autocorrelation in response
Spatial lag model

- Incorporates spatial effects by including a spatially lagged dependent variable as an additional predictor

$$y = \rho W y + x\beta + \varepsilon$$

- Where: Wy : the spatially lagged response for weights matrix W
- x : matrix of observations on the explanatory variables

  ε : vector of error terms

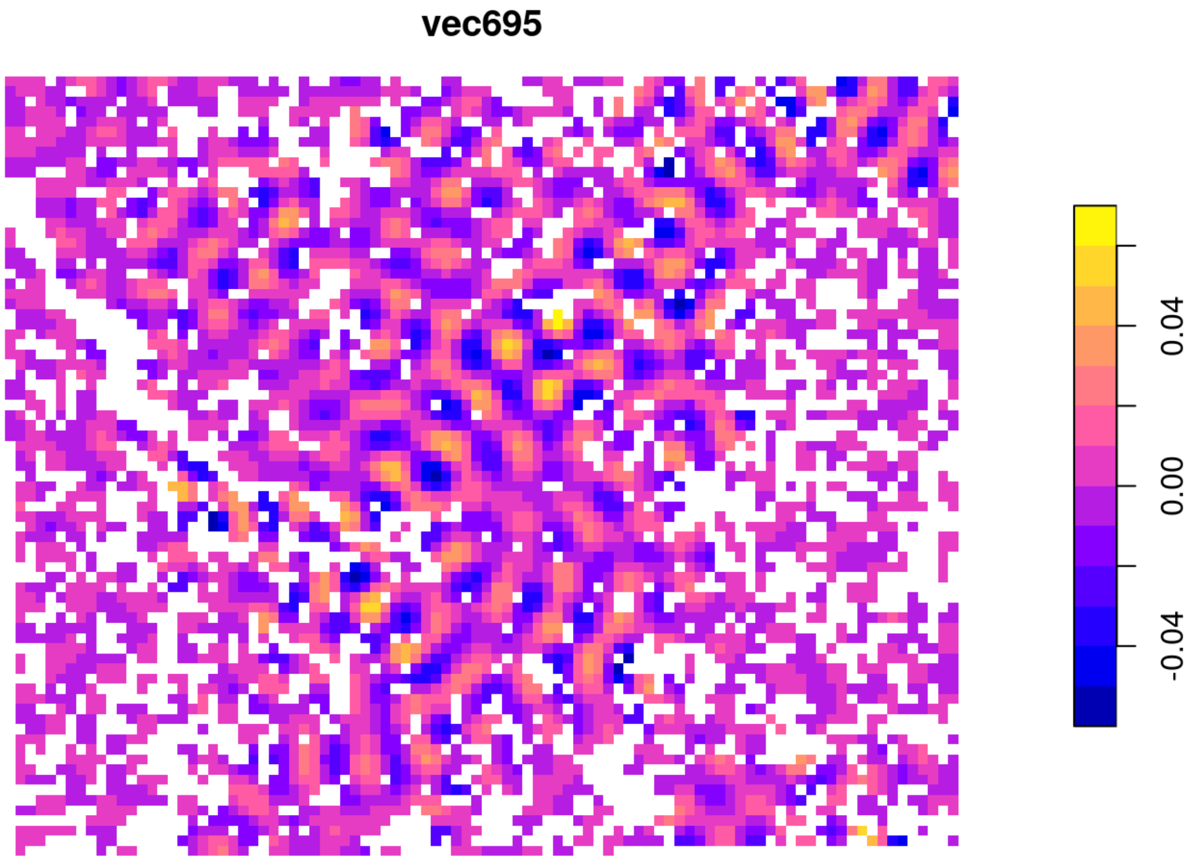
  ρ : spatial coefficient

- If there is no spatial dependence, and y does no depend on neighboring y values, $= 0$

Spatial eigenvector mapping

- Based on the eigenfunction decomposition of spatial connectivity matrices
- Eigenvectors from these matrices represent the decompositions the spatial weight Matrix into all mutually orthogonal m
- Eigenvectors with positive eigenvalues represent positive autocorrelation, whereas eigenvectors with negative eigenvalues represent negative autocorrelation
- Only eigenvectors with positive eigenvalues are used

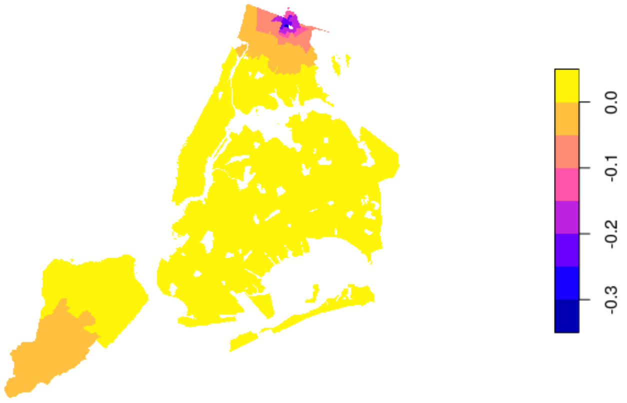
Spatial eigenvectos



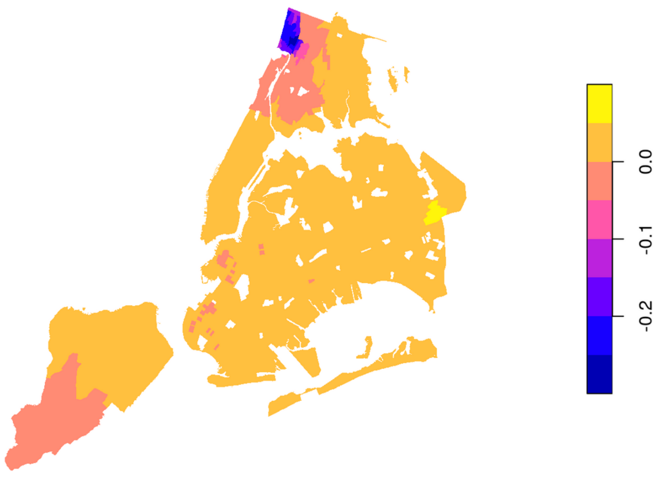
- Spatial pattern for independent dimensions of spatial structure
- Could be used to build hypothesis about missing covariates etc.

Spatial eigenvectos

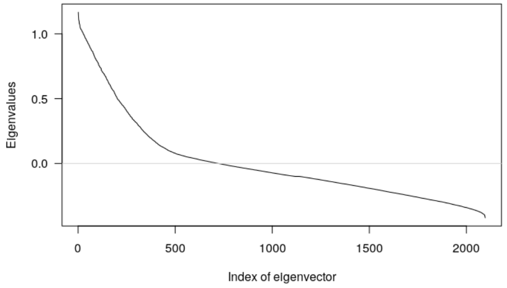
10nn, W: EV2



10nn, W: EV3



- Correlation length decreases with decreasing eigenvalue of the eigenvectors



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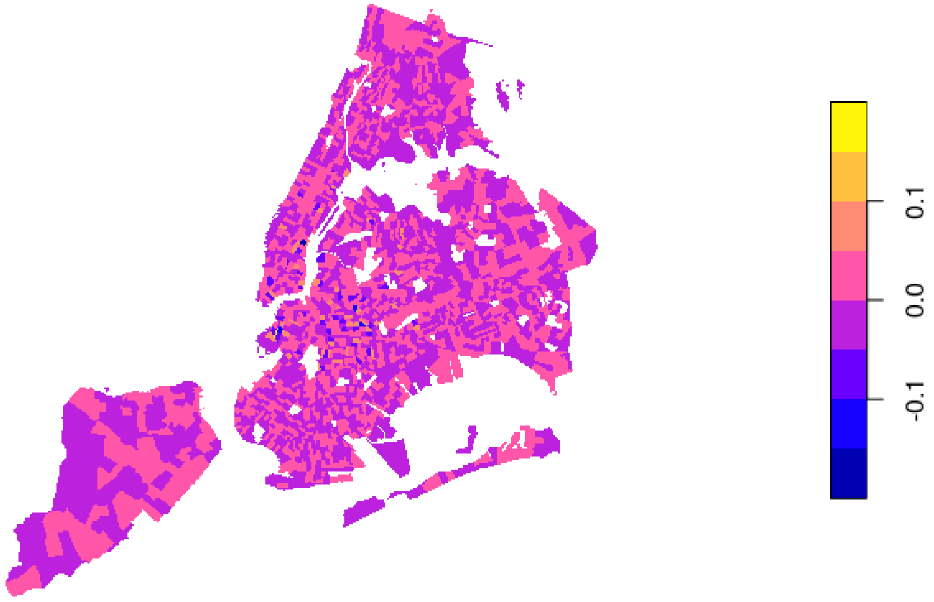
10nn, W: EV311

Spatial eigenvectos

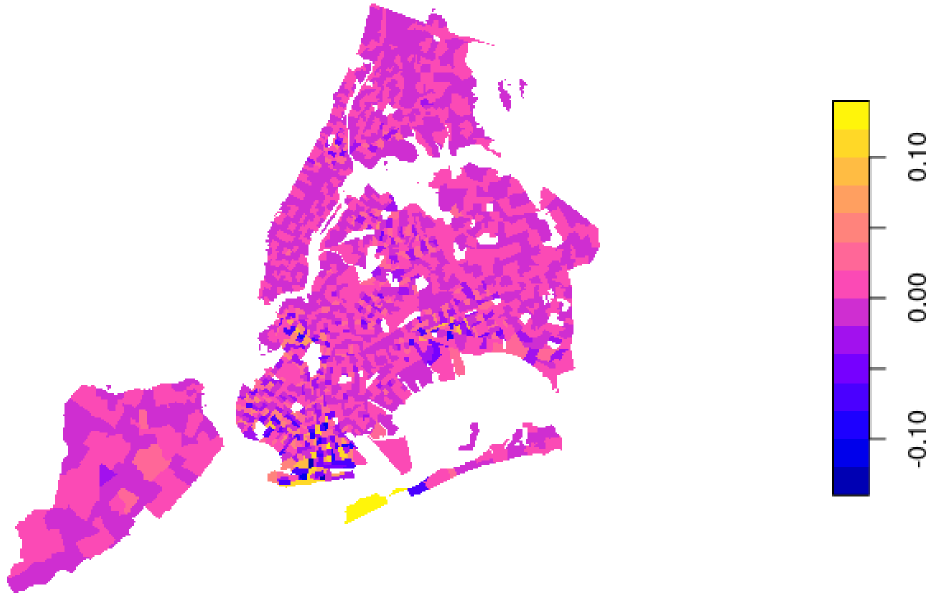
- Eigenvectors with negative eigenvalue represent negative spatial autocorrelation

10nn, W: 1000



...

10nn, W: 2000



Spatial eigenvector mapping

- Compute connectivity matrix
  - Needs to be symmetric!
- Compute eigenvectors of the centered similarity matrix
- Select eigenvectors to be included in the GLM
  - eigenvectors are added to a model until the spatial autocorrelation in the residuals, measured by Moran’s I, is non-significant

Literature for further studies

- Anselin, L., Rey, S.J., 2014. Modern Spatial Econometrics in Practice. GeoDa Press LLC, Chicago, IL.
  - Spatial lag and error model
  - Not with R but GeoDa, GeoDaSpatial and Python
  - Packed with Theory and Examples
- Bivand, R.S., Pebesma, E.J., Gómez-rubio, V., 2008. Applied Spatial Data Analysis with R. Springer, New York, NY.
  - Overview about spatial tasks in R
  - A bit geeky, not much theory
- Fortin, M.-J., Dale, M., 2005. Spatial analysis: a guide for ecologists. Cambridge University Press, Cambridge (UK).
  - Excellent overview about spatial statistics
  - No code

Literature for further studies

- Dormann et al. 2007. Methods to account for spatial autocorrelation in the analysis of species distributional data: a review. Ecography 30:609–628.
  - Excellent overview about methods
  - R code
  - Strengths ad weaknesses of the different methods
- Carl, G., C. F. Dormann, and I. Kühn. 2008. A wavelet-based method to remove spatial autocorrelation in the analysis of species distributional data. Web Ecology:22–29
  - Add on to Dormann et al (2007)
  - R code
- Griffith, D. A. 2006. Spatial Modelling in Ecology: The Flexibility of Eigenfunction Spatial Analysis. Ecology 87:2603–2613.
  - Spatial eigenvector mapping and filtering

Literature for further studies

- Bivand, R., Piras, G., 2015. Comparing Implementations of Estimation Methods for Spatial Econometrics. J. Stat. Softw. 63, 1–36. doi:10.18637/jss.v063.i18
  - Comparison of the different implementations of the spatial lag and error model in different software packages
  - Good theoretical overview
- Vignettes for the different spatial R packages
- Spatial task view on CRAN

Literature for further studies

- Chun, Y. & Griffith, D. A. (2013): Spatial Statistics & Geostatistics, SAGE
- Griffith, D.A., Chun, Y. and Li, B. (2019): Spatial Regression Analysis Using Eigenvector Spatial Filtering, Elsevier Academic Press
- Haining & Li (2020) Modellign Spatial and Spatio-Temporal Data – A Bayesian Approach, CRC Press